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SHORT COMMUNICATION

Analytical solutions of couple stress fluid flows with slip boundary conditions

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Abstract In the present article, the exact solutions for fundamental flows namely Couette, Poiseuille and generalized Couette flows of an incompressible couple stress fluid between parallel plates are obtained using slip boundary conditions. The effect of various parameters on velocity for each problem is discussed. It is found that, for each of the problems, the solution in the limiting case as couple stresses approaches to zero is similar to that of classical viscous Newtonian fluid. The results indicate that, the presence of couple stresses decreases the velocity of the fluid.

1. Introduction

The flows of non-Newtonian fluids have many practical applications in modern technology and industries, led various researchers to attempt diverse flow problems related to several non-Newtonian fluids. One such fluid that has attracted the attention of numerous researchers in fluid mechanics during the last five decades is the theory of couple stress fluids proposed by Stokes [1]. Couple stress fluid theory is a simple generalization of the classical theory of viscous Newtonian fluids that allow the sustenance of couple stresses and body couples in the fluid medium. The concept of couple stresses arises due to the way in which the mechanical interactions in the fluid medium are modeled. The stress tensor here is not symmetric. This theory adequately describes the flow behavior of fluids containing a substructure such as lubricants with polymer additives, liquid crystals and animal blood [2,3]. Many studies have been made on the hydrodynamic lubrication of squeeze film flows considering the lubricant as a couple stress fluid and the studies revealed that the couple stress fluid increases the load carrying capacity of the journal bearing [4–7].

A majority of flows of Newtonian and non-Newtonian fluids have been studied under no-slip boundary condition. However, this condition might not always hold, and that the fluid slippage might occur at the solid boundaries [8–11]. The works of O'Neill et al. [12] and Basset [13] indicate the existence of slip at the solid boundary. In fact, in early 19th century Navier [14] proposed a general boundary condition that presents the possibility of slip at the solid boundary. This boundary condition assumes that the tangential velocity of the fluid

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relative to the solid at a point on its surface is proportional to the tangential stress acting at that point. A review of experimental studies regarding the slip of Newtonian fluids at solid interface is given by Neto et al. [15].

The study of creeping flows of non-Newtonian fluids has gained increasing interest due to its applications in engineering and industry such as material processing in chemical engineering and hydraulic fracturing in oil recovery [16–18]. The pressure driven flow or Poiseuille flow is one of the most commonly encountered creeping flow, which has enormous applications in polymer processing such as extraction and die flow, injection molding, blow molding and asthenospheric flows [19]. Although pressure driven flows are unidirectional, and have been studied earlier for Newtonian and some non-Newtonian fluids, they still attract special attention in number of emerging problems [20–23]. In view of this, several researchers have studied the pressure driven flows for diverse non-Newtonian fluids. Ellahi [11] has examined the effect of the slip condition on flow of an Oldroyd 8-constant fluid in a channel. Yang and Zhu [24] have analyzed the squeeze flow of Bingham fluid in the small gap between parallel disks with slip boundary condition. Chen and Zhu [25] obtained the analytical solution of Couette–Poiseuille flow of Bingham fluids between two porous parallel plates with slip conditions. Hron et al. [26] established closed form analytical solution for the flows of incompressible non-Newtonian fluids with Navier's slip conditions at the boundary. Hayat et al. [27] discussed the effect of the slip condition on the flows of an Oldroyd 6-constant fluid between parallel plates. Abelman et al. [28] have analyzed the steady rotating flow of incompressible third grade fluid due to suddenly moved lower plate with partial slip of the fluid on the plate. Ellahi et al. [29] found the exact solutions for three fundamental flows namely Couette, Poiseuille and generalized Couette flows, under nonlinear slip conditions. Ferrás et al. [30] presented analytical solutions for both Newtonian and inelastic non-Newtonian fluids with slip boundary conditions in Couette and Poiseuille flows. Khaled and Vafai [31] have obtained the exact solutions of Stokes and Couette flows due to an oscillating wall with slip conditions. To the best knowledge of the authors, the Poiseuille, Couette and generalized Couette flows of couple stress fluid between parallel plates have not been solved subject to slip boundary conditions.

The aim of present communication is to establish the analytical solutions for the three classical flow problems namely *Poiseuille, Couette and generalized Couette flows* of an incompressible couple stress fluid between parallel plates under slip boundary conditions. The slip boundary conditions are applied at the boundaries of upper and lower plates. Three cases have been discussed: In the first case it is assumed that the lower and upper plates are moving with different translatory constant velocities assuming the pressure is constant (the resultant flow is known as Couette flow), in the second case both lower and upper plates are at rest and the flow is due to the constant pressure gradient which is also known as Poiseuille flow while in the last case it is assumed that the flow is generated due to the translatory motion of the upper plate when the lower plate is at rest and simultaneously a constant pressure gradient is applied. The flow in the last case known to be generalized Couette flow or Couette–Poiseuille flow. The paper is organized in terms of four sections. The next section is devoted for presenting the formulation of the basic equations governing the flow of couple stress fluid in Cartesian coordinates for the presently considered flows followed by their solutions. The results are discussed in Section 3 and the concluding remarks are presented in the last section.

2. Governing equations, problem formulation and their solutions

The equations governing the flow of a couple stress fluid are given by [2]

$$\frac{d\rho}{dt} + \rho\nabla \bullet \bar{q} = 0 \tag{2.1}$$

$$\rho\frac{d\bar{q}}{dt} = \rho\bar{f} + \frac{1}{2}\nabla \times (\rho\bar{c}) - \nabla p - \mu\nabla \times \nabla \times \bar{q} - \eta\nabla \times \nabla \times \nabla \times \nabla \times \bar{q} + (\lambda_1 + 2\mu)\nabla(\nabla \bullet \bar{q}) \tag{2.2}$$

where $\bar{q}$ and ρ are the velocity and the density of the fluid respectively, p is the fluid pressure at any point, $\bar{f}$ and $\bar{c}$ are the body force per unit mass and body couple per unit mass respectively.

The constitutive equation connecting the force stress tensor t_{ij} and rate of deformation tensor d_{ij} is given by,

$$t_{ij} = (-p + \lambda_1\nabla \bullet \bar{q})\delta_{ij} + 2\mu d_{ij} - \frac{1}{2}\varepsilon_{ijk}[m_{,k} + 4\eta\omega_{k,rr} + \rho c_k]$$

The couple stress tensor m_{ij} that arises in the theory has the linear constitutive relation

$$m_{ij} = \frac{1}{3}m\delta_{ij} + 4\eta'\omega_{j,i} + 4\eta\omega_{i,j}$$

In the above $\omega = \frac{1}{2}\nabla \times \bar{q}$ is the spin vector, $\omega_{i,j}$ is the spin tensor, d_{ij} is the rate of deformation tensor, m is the trace of couple stress tensor m_{ij} and ρc_k is the body couple vector. The quantities λ_1 and μ are the viscosity coefficients and η and η' are the couple stress viscosity coefficients. These material constants are constrained by the inequalities,

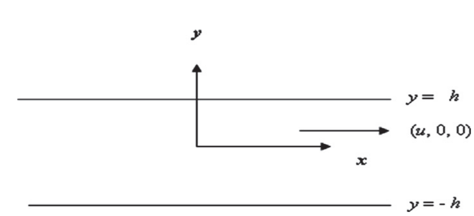
$$\mu \geqslant 0;\ 2\mu + 3\lambda_1 \geqslant 0;\ \eta \geqslant 0;\ |\eta'| \leqslant \eta$$

In the absence of body couples, the field equations governing the flow of an incompressible couple stress fluid are

$$\nabla \bullet \bar{q} = 0 \tag{2.3}$$

$$\rho\frac{d\bar{q}}{dt} = \rho\bar{f} - \nabla p - \mu\nabla \times \nabla \times \bar{q} - \eta\nabla \times \nabla \times \nabla \times \nabla \times \bar{q} \tag{2.4}$$

where $\bar{q}$ and ρ are the velocity and the density of the fluid respectively, p is the fluid pressure at any point and $\bar{f}$ is the body force per unit mass. μ and η are respectively viscosity and couple stress viscosity coefficients.



Flow Configuration

For unidirectional steady flow between parallel plates, the velocity field $\bar{q} = (u(y), 0, 0)$ and it automatically satisfies the continuity Eq. (2.3). The momentum Eq. (2.4) governing the flow, in the absence of body forces, reduces to