Alexandria Engineering Journal (2014) 53, 737–745

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

FACULTY OF ENGINEERING ALEXANDRIA UNIVERSITY

ORIGINAL ARTICLE

Nanofluid flow over an unsteady stretching surface in presence of thermal radiation

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Received 27 December 2013; revised 28 April 2014; accepted 14 May 2014
Available online 14 June 2014

KEYWORDS

Nanofluid;
Heat and mass transfer;
Stretching sheet;
Thermal radiation

Abstract This paper investigates the unsteady boundary layer flow of a nanofluid over a heated stretching sheet with thermal radiation. The transport model employed includes the effects of Brownian motion and thermophoresis. The unsteadiness in the flow field is caused by the time-dependence of the stretching velocity, free stream velocity and the surface temperature. The unsteady boundary layer equations are transformed to a system of non-linear ordinary differential equations and solved numerically using a shooting method together with Runge–Kutta–Fehlberg scheme. The clear liquid results from this study are in agreement with the results reported in the literature. It is found that the heat transfer rate at the surface increases in the presence of Brownian motion but reverse effect occurs for thermophoresis.

1. Introduction

Stagnation point flow of an incompressible viscous fluid induced by a stretching sheet has important practical applications in many industries, such as the aerodynamics extrusion of plastic sheets, the cooling of metallic plate, continuous stretching of plastic films and artificial fibers. Extensive research on boundary layer flow over a stretching surface has been under taken since pioneering work of Sakiadis [1] because of its engineering applications. Crane [2] obtained a closed form solution for a stretching sheet whose velocity is proportional to the distance from the slit. Further, Weidman and Magyari [3], Chen and Char [4], Dutta et al. [5] etc. have studied various aspects of the problem of a stretching sheet in its own plane and of the stagnation point flow toward a stretching sheet. The studies mentioned above dealt with the steady flows, but many problems of practical interest may be unsteady. The unsteadiness is due to the change in the stretching velocity, free stream velocity or wall temperature etc. The mechanical and thermal characteristics of such an unsteady process were investigated both analytically and numerically in the boundary layer approximation, assuming a linear variation in the steady stretching velocity with the longitudinal

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

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http://dx.doi.org/10.1016/j.aej.2014.05.002

coordinate and an inverse linear law for its decrease with time during the gradual switch-off process. Various aspects of the unsteady stretching sheet problem have been investigated by many authors (cf. [6–10]).

The effects of thermal radiation on the flow and heat transfer have important applications in physics and engineering, especially, in space technology and high temperature processes. Thermal radiation effects may also play an important role in controlling heat transfer in industry where the quality of the final product depends on the heat controlling factors to some extent. The effects of radiation on heat transfer problems have studied by Raptis [11], Makinde [12], Ibrahim et al. [13], Hayat et al. [14], Pal [15], Shit and Haldar [16]. Recently, Das [17] investigated the impact of thermal radiation on MHD slip flow over a flat plate with variable fluid properties.

Considerable efforts have been directed toward the study of the boundary layer flow and heat transfer over a stretching sheet because of its numerous industrial applications such as electronic, power, manufacturing, aerospace and transportation industries. Common heat transfer fluids such as water, ethylene glycol, toluene and engine oil have limited heat transfer capabilities due to their low heat transfer properties. In contrast, metals have higher thermal conductivities than these fluids. Therefore, it is desirable to combine the two substances to produce a heat transfer medium that behaves like a fluid but has the higher heat transfer properties. The term nanofluid refers to a liquid suspension containing tiny particles having diameter less than 50 nm. Choi [18] experimentally verified that addition of small amount of nanoparticles appreciably enhances the effective thermal conductivity of the base fluid. The common nanoparticles that have been used are aluminum, copper, iron and titanium or their oxides. Various benefits of the application of nanofluids include the following: improved heat transfer, heat transfer system size reduction, micro-channel cooling and miniaturization of the system. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [19] who considered seven slip mechanisms that can produce a relative velocity between nanoparticles and the base fluid. Of all these mechanisms, only Brownian diffusion and thermophoresis were found to be important. An excellent assessment of nanofluid physics and developments has been provided by Das et al. [20] and Eastman et al. [21]. The influences of nanoparticles on natural convection boundary layer flow past a vertical plate by taking Brownian motion and thermophoresis into account was investigated by Kuznetsov and Nield [22]. Godson et al. [23] presented an overview on experimental and theoretical studies on convective heat transfer in nanofluids and their applications.

Akyildiz et al. [24] discussed nanoboundary layer fluid flows over stretching surfaces. Chamkha et al. [25] investigated the mixed convection flow of a nanofluid past a stretching surface in the presence of Brownian motion and thermophoresis effects. Das [26] studied Lie group analysis of stagnation-point flow of a nanofluid. Nanofluid flow over a shrinking sheet in the presence of surface slip was discussed by Das [27]. Recently heat transfer analysis of nanofluid over an exponentially stretching sheet was investigated by Nadeem et al. [28].

The objective of the present work was to study the effect of thermal radiation on boundary layer flow of a nanofluid over a heated stretching sheet with an unsteady free stream condition. Numerical results are obtained using a shooting technique together with Runge–Kutta–Fehlberg schemes. The paper is organized as follows. Mathematical analysis regarding problem formulation is presented in Section 2. Section 3 comprises the method of solution and code verification. Discussion related to plots is presented in Section 4. Section 5 lists the main observations.

2. Mathematical analysis

2.1. Governing equations

Considering the two-dimensional unsteady boundary layer flow of a nanofluid over a heated stretching sheet with thermal radiation, the coordinate system under consideration is such that x measures the distance along the sheet and y measures the distance normally into the fluid (Fig. 1). The flow is assumed to be confined to $y > 0$. Two equal and opposite forces are impulsively applied along the x-axis so that the sheet is stretched keeping to fixed origin. Let us consider that for time $t < 0$ the fluid and heat flows are steady. The unsteady fluid and heat flows start at $t = 0$, the sheet being stretched with the velocity $U_w(x, t)$ along the x-axis. It is also assumed that the ambient fluid is moved with a velocity $U_e(x, t)$ in the y-direction toward the stagnation point on the plate. The temperature of the sheet $T_w(x, t)$ and the value of nanoparticle volume fraction $C_w(x, t)$ at the surface vary both with the distance x along the sheet and time t and higher than the ambient temperature T_∞ and concentration C_∞ respectively. In view of thermal equilibrium, there is no slip between the base fluid and suspended nanoparticles. Since the velocity of the nanofluid is low (laminar flow), the viscous dissipative heat is assumed to be negligible here.

Under the above assumptions, the basic unsteady conservation of mass, momentum, thermal energy equations of nanofluid and nanoparticle fraction in the presence of thermal radiation past a stretching sheet can be expressed as follows (see Refs. [6,8,26]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U_e}{\partial t} + U_e\frac{\partial U_e}{\partial x} + \nu_f\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right] - \frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y} \tag{3}$$

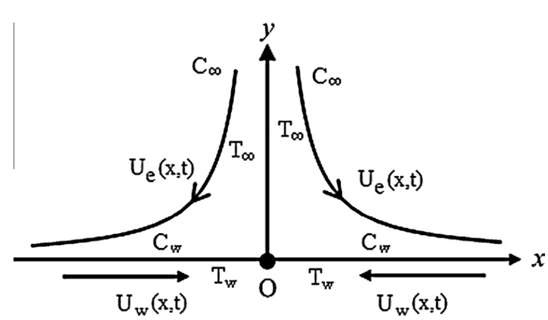

Figure 1 Physical model and coordinate system.