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ORIGINAL ARTICLE

The boundary layer flow of hyperbolic tangent fluid over a vertical exponentially stretching cylinder

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Abstract The present problem is the steady boundary layer flow and heat transfer of a hyperbolic tangent fluid flowing over a vertical exponentially stretching cylinder in its axial direction. After applying usual boundary layer with a suitable similarity transformation to the given partial differential equations and the boundary conditions, a system of coupled nonlinear ordinary differential equations is obtained. This system of ordinary differential equations subject to the boundary conditions is solved with the help of Runge–Kutta–Fehlberg method. The effects of the involved parameters such as Reynolds numbers, Prandtl numbers, Weissenberg numbers and the natural convection parameter are presented through the graphs. The associated physical properties on the flow and heat transfer characteristics that is the skin friction coefficient and Nusselt numbers are presented for different parameters.

1. Introduction

During the past many years, number of researchers has worked on non-Newtonian fluids. Wang [1] analyzed non-Newtonian fluids for mixed convection heat transfer from a vertical plate. Xu et al. [2] have presented the series solutions of unsteady boundary layer flows of non-Newtonian fluids near a forward stagnation point. Ellahi and Afzal [3] have discussed the effects of variable viscosity on a third grade fluid with porous medium. In another paper Ellahi [4] has presented the effects of MHD (Magneto hydrodynamics) and temperature dependent viscosity on the flow of non-Newtonian nanofluid in a pipe. Nadeem et al. [5] analyzed the non-orthogonal stagnation point flow of a nano non-Newtonian fluid toward a stretching surface with heat transfer. Labropulu et al. [6] wrote an article on non-orthogonal stagnation-point flow toward a stretching surface in a non-Newtonian fluid with heat transfer.

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An important branch of the non-Newtonian fluid models is the hyperbolic tangent fluid model. The hyperbolic tangent fluid is used extensively for different laboratory experiments. Friedman et al. [7] have used the hyperbolic tangent fluid model for large-scale magneto-rheological fluid damper coils. Recently, Nadeem and Akram [8] jointly published a paper on peristaltic transport of a hyperbolic tangent fluid model in an asymmetric channel. In another paper Nadeem and Akram [9] have presented the effects of partial slip on the peristaltic transport of a hyperbolic tangent fluid model in an asymmetric channel. Only few researchers have worked on different non-Newtonian fluid models [10–17]. Ali [18] analyzed the heat transfer characteristics of a continuous stretching surface. Ishak et al. [19] investigated the uniform suction/blowing effect on flow and heat transfer due to a stretching cylinder. Wang [20] studied the natural convection on a vertical stretching cylinder. Some notable studies related to the current topic can be found at [21–25].

However, up to now to the best of our knowledge no paper is published on boundary layer flow of a hyperbolic tangent fluid flowing over a vertical exponentially stretching cylinder. The governing boundary layer equations of hyperbolic tangent fluid model are presented and solved numerically subject to the boundary conditions of exponentially stretching cylinder. The physical behavior of the useful parameters is discussed through graphs and tables. The expression for coefficient of skin friction and local Nusselt number are computed numerically.

2. Fluid model

For the hyperbolic tangent fluid the continuity and momentum equations are given as

$$\operatorname{div} V = 0,$$

$$\rho \frac{dV}{dt} = \operatorname{div} \tau + \rho b,$$

where ρ is the density, V is the velocity vector, τ is the Cauchy stress tensor, b represents the specific body force vector and d/dt represents the material time derivative. The constitutive equations for hyperbolic tangent fluid model is given by [8]

$$S = -pI + \tau,$$

$$\tilde{\tau} = -[\eta_\infty + (\eta_0 + \eta_\infty)\tanh(\Gamma\bar{\dot{\gamma}})^n]\bar{\dot{\gamma}}$$

$$\text{where}\ \bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i\sum_j \overline{\dot{\gamma}_{i,j}\dot{\gamma}_{ji}}} = \sqrt{\frac{1}{2}\prod}$$

$$\text{where}\ \prod = \frac{1}{2} trac[\operatorname{grad} V + (\operatorname{grad} V)^t]^2$$

We consider the case for which $\eta_\infty = 0$ and $\Gamma\,\bar{\dot{\gamma}} < 1$. The component of extra stress tensor, therefore, can be written as

$$\tilde{\tau} = -\eta_0(\Gamma\bar{\dot{\gamma}})^n\bar{\dot{\gamma}}$$

$$\tilde{\tau} = -\eta_0[1 + \Gamma\bar{\dot{\gamma}} - 1]^n\bar{\dot{\gamma}}$$

$$\tilde{\tau} = -\eta_0[1 + n[\Gamma\bar{\dot{\gamma}} - 1]\dot{\gamma}$$

3. Formulation

Consider the problem of natural convection boundary layer flow of a hyperbolic tangent fluid flowing over a vertical circular cylinder of radius a. The cylinder is assumed to be stretched exponentially along the axial direction with velocity U_w. The temperature at the surface of the cylinder is assumed to be T_w and the uniform ambient temperature is taken as T_∞. Under these assumptions the boundary layer equations of motion and heat transfer are

$$u_r + \frac{u}{r} + w_z = 0, \tag{1}$$

$$uw_r + ww_z = g\beta(T - T_\infty) + \nu\left[(1-n)\left(w_{rr} + \frac{1}{r}w_r\right) + \frac{n\Gamma w_r}{2}\left(2w_{rr} + \frac{1}{r}w_r\right)\right] \tag{2}$$

$$uT_r + wT_z = \alpha\left(T_{rr} + \frac{1}{r}T_r\right), \tag{3}$$

where the velocity components along the (r, z) axes are (u, w), ρ is density, ν is the kinematic viscosity, p is pressure, g is the gravitational acceleration along the z-direction, β is the coefficient of thermal expansion, T is the temperature, η_∞ is the infinite shear rate viscosity, η_0 is the zero shear rate viscosity, Γ is the time constant, n is the power law index, and α is the thermal diffusivity. The corresponding boundary conditions for the problem are

$$u(a,z) = 0,\ w(a,z) = U_w\ w(r,z) \to 0 \text{ as } r \to \infty, \tag{4}$$

$$T(a,z) = T_w(z),\ T(r,z) \to T_\infty \text{ as } r \to \infty, \tag{5}$$

where $U_w = 2ake^{z/a}$ (k is dimensional constant) is the fluid velocity at the surface of the cylinder.

4. Solution of the problem

Introduce the following similarity transformations:

$$u = -\frac{1}{2}U_w\frac{f(\eta)}{\sqrt{\eta}}, \quad w = U_w f'(\eta), \tag{6}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \frac{r^2}{a^2}, \tag{7}$$

where the characteristic temperature difference is calculated from the relations $T_w - T_\infty = T_0 e^{z/a}$. With the help of transformations (6) and (7), Eqs. (1)–(3) take the form

$$2(1-n)(\eta f''' + f'') + nF\sqrt{\eta}f''(4\eta f''' + 3f'') + \mathrm{Re}(ff'' - f'^2) + Re\lambda\theta = 0, \tag{8}$$

$$\eta\theta'' + \theta' + \mathrm{RePr}(f\theta' - f'\theta) = 0, \tag{9}$$

in which $\lambda = g\beta a(T_w - T_\infty)/U_w^2$ is the natural convection parameter, $\Pr = \nu/\alpha$ is the Prandtl number $We = 4\Gamma U_w/a$ is the Weissenberg number and $Re = aU_w/4\nu$ is the Reynolds number. The boundary conditions in nondimensional form become

$$f(1) = 0,\ f'(1) = 1,\ f' \to 0, \text{ as } \eta \to \infty, \tag{10}$$

$$\theta(1) = 1,\ \theta \to 0, \text{ as } \eta \to \infty. \tag{11}$$