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Random Blume–Emery–Griffiths model on the Bethe lattice

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ABSTRACT

The random phase transitions of the Blume–Emery–Griffiths (BEG) model for the spin-1 system are investigated on the Bethe lattice and the phase diagrams of the model are obtained. The biquadratic exchange interaction (K) is turned on, i.e. the BEG model, with probability p either attractively ($K > 0$) or repulsively ($K < 0$) and turned off, which leads to the BC model, with the probability $(1 - p)$ throughout the Bethe lattice. By taking the bilinear exchange interaction parameter J as a scaling parameter, the effects of the competitions between the reduced crystal fields (D/J), reduced biquadratic exchange interaction parameter (K/J) and the reduced temperature (kT/J) for given values of the probability when the coordination number is $q=4$, i.e. on a square lattice, are studied in detail.

1. Introduction

After the introduction of the Blume–Capel (BC) model with J and D interactions [1], a few years later the BEG model [2] was introduced by adding the interaction parameter K into the Hamiltonian in order to study the phase separation and superfluid ordering in ^{3}He–^{4}He mixtures. There is a vast literature about these two models, therefore, only the ones which may be related to this work are to be mentioned here. It is well-known that the spin-1 BC model gives second- and first-order phase transition lines meeting at the tricritical points, see for example [3]. The addition of K enriches the phase diagrams, since now K also gets involved into the competitions between J, D and temperature in the possible phase transitions of the system. The BEG model was studied for many different physical reasonings on the spin-1 systems by using many different theoretical methods by using the mean field theory (MFT) [4–7], by the use of the effective field theory [8–12], in terms of the cluster variation method [13–24], within the Monte Carlo simulations [25–31], on the Bethe lattice [32–34], within the framework of a finite cluster theory [35], within a functional integration approach [36], by using the linear chain approximation [37], by using the replica method [38], on the cellular automaton [39], upon transforming the spin model into an equivalent fermionic model [40], through a real-space renormalization-group approach and a MFT [41,42], in the Migdal–Kadanoff renormalization group [43], using the real-space renormalization group procedure [44] and by using the MFT, transfer-matrix calculations and position-space renormalization-group calculations [45].

In order to randomize the effects of the interaction parameters for a given system, some distribution functions are used. The Gaussian, bimodal or trimodal types of the distributions are commonly used in the literature in order to randomize the effects of the longitudinal or transverse magnetic field, crystal field and bilinear interaction couplings. This randomization deeply affects the critical behaviors of the system and so far it is only applied for the above mentioned system parameters for the spin-1 BEG model to the best of our knowledge. In this work, we consider the randomness of the biquadratic exchange interaction K in a bimodal form by turning it on with probability p for either ($K > 0$) or ($K < 0$) values, attractive and repulsive cases, i.e. $p=1$ corresponds to the BEG model, or by turning it off with probability $(1 - p)$, which reduces to the BC model when $p=0$. For the other values of p, the model becomes the random BEG model. Instead of studying the model on real lattice systems, we use the Bethe lattice which is a fictitious tree, i.e. a connected graph without circuits, and

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historically it gets its name from the fact that its partition function is exactly that of an Ising model in the Bethe approximation [46]. The Cayley tree and the Bethe lattice have been widely used in solid state and statistical physics.

Therefore, we carry out our calculations on the Bethe lattice for the spin-1 model with the number of the nearest-neighbors (NN), i.e. coordination number, $q=4$ by using the exact recursion relations [47]. The phase diagrams are calculated exactly on the (kT/J, D/J) and (kT/J, K/J) planes for given D/J and K/J values, respectively, with $0 \le p \le 1$ when $q=4$.

The rest of this work is organized as follows: the formulation of the problem is presented on the Bethe lattice in terms of the recursion relations in Section 2.

2. Formulation and bimodal random distribution

The Hamiltonian for the spin-1 BEG model may be given as

$$\mathcal{H} = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - \sum_{\langle ij\rangle}K_{ij}\sigma_i^2\sigma_j^2 - D\sum_i \sigma_i^2, \tag{1}$$

where σ_i is a spin-1 parameter with the values of ± 1 and 0, J is the bilinear exchange interaction parameter, D is the crystal field and K_{ij} is the site-dependent biquadratic exchange interaction parameter between the NN spins which may be taken either attractive ($K>0$) or repulsive ($K<0$). K is assumed to be distributed throughout the Bethe lattice with probabilities p and $1-p$ for $K>0$ or $K<0$ and $K=0$, respectively, as

$$P(K_{ij}) = p\delta(K_{ij}-K) + (1-p)\delta(K_{ij}). \tag{2}$$

where the case of $p=1$ corresponds to the regular BEG model and the case of $p=0$ is the BC model. The model becomes the random BEG model for the intermediate values of p, i.e. when $0<p<1$.

In order to obtain the phase diagrams of the model, one has to obtain the order-parameters. Since the spin-1 model is a three state and two order-parameter system, the order parameters are the magnetization and the quadrupolar moments. In order to obtain their exact calculation on the Bethe lattice, one starts from a central spin σ_0 which may be called the first generation spin. σ_0 has q NN's, which form the second generation spins σ_1. Each spin in the second generation is joined to $(q-1)$ NN's. Therefore, in total, the second generation has $[q(q-1)]$ NN's which form the third generation spins σ_2 and so on to infinity. Each generation of spins is called a shell of the Bethe lattice and the number of generations goes to infinity in the thermodynamic limit, i.e $n\to\infty$. In order to obtain the order parameters, one usually starts with the partition function which is defined as

$$Z = \sum_{All\ Config.} e^{-\beta\mathcal{H}} = \sum_{Spc} P(Spc), \tag{3}$$

where $P(Spc)$ can be thought of as an unnormalized probability distribution and $\beta = 1/(kT)$, with k being the Boltzmann constant and T being the absolute temperature.

Instead of giving all the details of the calculation in terms of the recursion relations, we refer readers, for example, to reference [33]. The recursion relations are given as the ratios of the partial partition functions, g_n, and are calculated as

$$X_n^{(ij)} = \frac{g_n(+1)}{g_n(0)} = \frac{e^{\beta(J+D+K_{ij})}[X_{n-1}^{(ij)}]^{q-1} + e^{\beta(-J+D+K_{ij})}[Y_{n-1}^{(ij)}]^{q-1} + 1}{e^{\beta D}[X_{n-1}^{(ij)}]^{q-1} + e^{\beta D}[Y_{n-1}^{(ij)}]^{q-1} + 1},$$
$$Y_n^{(ij)} = \frac{g_n(-1)}{g_n(0)} = \frac{e^{\beta(-J+D+K_{ij})}[X_{n-1}^{(ij)}]^{q-1} + e^{\beta(J+D+K_{ij})}[Y_{n-1}^{(ij)}]^{q-1} + 1}{e^{\beta D}[X_{n-1}^{(ij)}]^{q-1} + e^{\beta D}[Y_{n-1}^{(ij)}]^{q-1} + 1}. \tag{4}$$

Note that one has to carry out an averaging procedure over the biquadratic interaction distribution $P(K_{ij})$ to get the correct recursion relations for the random BEG model, that is

$$X_n = \int X_n^{(ij)}P(K_{ij})\,dK_{ij} = \int X_n^{(ij)}[p\delta(K_{ij}-K) + (1-p)\delta(K_{ij})]\,dK_{ij},$$
$$Y_n = \int Y_n^{(ij)}P(K_{ij})\,dK_{ij} = \int Y_n^{(ij)}[p\delta(K_{ij}-K) + (1-p)\delta(K_{ij})]\,dK_{ij}. \tag{5}$$

After having obtained the recursion relations including the effects of randomization of K, we are now ready to present the formulations of the order-parameters in terms of the recursion relations. Since all sites of the BL are equivalent deep inside, one can pick a central spin, σ_0, and calculate its order parameters accordingly. The magnetization or dipolar moment for the central spin is given as

$$M = \langle\sigma_0\rangle = \frac{e^{\beta D}X_n^q - e^{\beta D}Y_n^q}{e^{\beta D}X_n^q + e^{\beta D}Y_n^q + 1}, \tag{6}$$

and similarly the quadrupolar moment for the central spin is found as

$$Q = \langle\sigma_0^2\rangle = \frac{e^{\beta D}X_n^q + e^{\beta D}Y_n^q}{e^{\beta D}X_n^q + e^{\beta D}Y_n^q + 1}. \tag{7}$$

In order to write these equations in terms of the reduced quantities, one simply sets $J=1$ and replaces D by D/J, K by K/J and $\beta = 1/kT$ by $\beta J = J/kT$. In the next section, we present the phase diagrams of the model obtained from the analysis of the thermal variations of the order-parameters.

3. Phase diagrams and findings

The phase diagrams of the random spin-1 BEG model are calculated by studying the thermal variations of the order-parameters and presented on the $(D/J, kT/J)$ and on the $(K/J, kT/J)$ planes for given values of K/J and D/J, respectively, for $0 \le p \le 1$ with $\Delta p = 0.1$ and $q=4$. The second-order phase transition lines are indicated by solid lines and are called simply T_c-lines while the first-order phase transition lines are shown by dashed lines and named T_t-lines. The tricritical points (TCP), i.e. the point at which the T_c- and T_t-lines meet, are shown by the filled circles. The ferromagnetic and paramagnetic phase regions are indicated by (F) and (P), respectively.

Before starting to present the figures, we should again note that the two limiting values of p, i.e. 0 and 1, lead to the BC and BEG models, respectively. So for $p=0$, K is turned off and the phase transition line is just the T_c- and T_t-lines combined at the TCP. The $p=1$ case is the well-known BEG model with the strongest K effect for either $K>0$ or $K<0$. For intermediate values of p, the model favors the BC model as $p\to 0$ and the regular BEG model when $p\to 1$. However, as we shall see in the figures, as soon as K is introduced into the system, the phase lines tend to become like the phase lines of the regular BEG model.

The phase diagrams obtained on the $(D/J, kT/J)$ planes for given values of $K/J = \pm 5.0, \pm 2.0, \pm 0.1$ and 1.0 are shown in Fig. 1. The critical lines for the case $K/J>0$ are always above the critical line of the BC case, i.e. they are seen at higher critical temperatures and higher negative D/J values than the values for $p=0$. These temperatures increase as p increases and are seen at higher negative D/J's for higher K/J's. For the repulsive case, they always stay inside the BC-line with lower critical temperatures and higher positive D/J values for higher p values. The temperatures of these lines decrease now as p increases and are found at higher D/J's for higher negative K/J's. The BC model gives a T_t-line at lower temperatures and at negative D/J values, thus we expect to have the same from the BEG model. As seen from the Hamiltonian, i.e. Eq. (1), K and D are the multiplication factors of the squares of the spin operators, so that they both affect the system similarly. When they are