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# Soliton excitations and stability in a square lattice model of ferromagnetic spin system

M.M. Latha*, T. Anitha

*Department of Physics, Women's Christian College, Nagercoil 629001, India*

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ABSTRACT

We investigate the nature of nonlinear spin excitations in a square lattice model of ferromagnetic (FM) spin system with bilinear and biquadratic interactions. Using the coherent state ansatz combined with the Holstein–Primakoff (HP) bosonic representation of spin operators, the dynamics is found to be governed by a discrete nonlinear equation which possesses soliton solution. The modulational instability aspects of the soliton excitations are analysed for small perturbations in wave vectors.



## 1. Introduction

Nonlinear spin excitations in magnetic materials have their applications in microwave communication systems and nonlinear processing devices where solitons play a role in the description of such excitations [1–11]. Several effective theoretical methods have been developed and applied to a large class of nonlinear excitations in magnets [12–28]. Of these, semiclassical approach turns out to be successful in the practical realization of a one dimensional spin system. Such systems have been studied extensively for manifestations of nonlinear spin dynamics [19–24]. In other words, the nature of solitary waves in a spin chain have been studied extensively and are implicit quite well, both in theoretical and experimental aspects. But the above consideration is not complete especially in a square lattice model. Recently, Zhong et al. [29,30] have studied experimentally the magnetic properties of the soft magnetic composite material *SOMALOY*$^{TM}$500 in a square sample under different patterns of flux density with 2D magnetic excitations. Recently, Latha et al. [31,32] have proposed a square lattice model of Heisenberg ferromagnetic spin system and studied soliton excitations after making a continuum approximation.

Additionally, the magnetic systems with a mixture of interactions act as vital dynamical models exhibiting fascinating nonlinear phenomena [7,10]. Among them, the biquadratic exchange interaction plays a major role and in existence there has been a substantial interest in the study of ferromagnetic spin chains with competing bilinear and biquadratic exchange interactions [33–36]. This has encouraged us to study soliton excitation in a square lattice model of ferromagnetic spin system with bilinear and biquadratic interactions.

Modulational instability (MI) is one of the most fundamental effects associated with wave propagation in nonlinear dispersive systems [37–40]. Very few studies on the stability aspects of discrete spin systems have been devoted to the 1D ferromagnetic systems. But so far, studies on the stability aspects of lattice FM spin system have not yet been reported in the literature. Having these in mind, in this paper, we also investigate the stability aspects of solitons in a square lattice model of Heisenberg spin system with bilinear and biquadratic interactions.

The plan of the paper is as follows. In Section 2, we present the Hamiltonian for a square lattice model of Heisenberg FM spin system with bilinear exchange and anisotropic interactions and construct the equation of motion in the semiclassical limit. In Section 3, we investigate the modulational instability aspects of the resulting equation by means of linear stability analysis. We study the nonlinear spin excitations by including biquadratic interactions in Section 4. In Section 5, we present the linear stability analysis and analyse the instability regions at different physical conditions and finally we provide the conclusions in Section 6.

* Corresponding author. Fax: +91 4652 228834.
*E-mail address:* lathaisaac@yahoo.com (M.M. Latha).

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## 2. Two dimensional Heisenberg spin chain with bilinear interactions

We consider for our study a square lattice model of Heisenberg Ferromagnetic spin system with bilinear and anisotropic interactions. The Hamiltonian for such a system is written as

$$\tilde{H} = -\sum_{n,m}[\tilde{J}(\vec{S}_{n,m}\cdot\vec{S}_{n+1,m}) + \tilde{J}_1(\vec{S}_{n,m}\cdot\vec{S}_{n,m+1}) + \tilde{J}_2(\vec{S}_{n,m}\cdot\vec{S}_{n+1,m+1}) - \tilde{A}(S^z_{n,m})^2], \tag{1}$$

where $\tilde{J}$ and $\tilde{J}_1$ correspond to the coefficients of bilinear exchange interactions along the $X$ and $Y$ directions respectively. $\tilde{J}_2$ refers to the neighbouring interaction along the diagonal. This bilinear exchange interaction is the cause for the parallel alignment of spin in ferromagnetic systems [41]. $\tilde{A}$ is the uniaxial crystal field anisotropic parameter. It constrains the spin lie in a plane perpendicular to the chain axis. $\vec{S}_{n,m} = (S^x_{n,m}, S^y_{n,m}, S^z_{n,m})$ represents the spin at the lattice site. In order to express the spin Hamiltonian in the dimensionless form, we write $\hat{S}_{n,m} = \frac{\vec{S}_{n,m}}{\hbar}$ and introduce $\hat{S}^{\pm}_{n,m} = \hat{S}^x_{n,m} \pm i\hat{S}^y_{n,m}$. In view of this, the dimensionless form of Hamiltonian (1) is written as

$$H = -\sum_{n,m}\Bigg[\frac{J}{2S^2}(\hat{S}^+_{n,m}\hat{S}^-_{n+1,m} + \hat{S}^-_{n,m}\hat{S}^+_{n+1,m} + 2\hat{S}^z_{n,m}\hat{S}^z_{n+1,m}) + \frac{J_1}{2S^2}(\hat{S}^+_{n,m}\hat{S}^-_{n,m+1} + \hat{S}^-_{n,m}\hat{S}^+_{n,m+1} + 2\hat{S}^z_{n,m}\hat{S}^z_{n,m+1}) + \frac{J_2}{2S^2}(\hat{S}^+_{n,m}\hat{S}^-_{n+1,m+1} + \hat{S}^-_{n,m}\hat{S}^+_{n+1,m+1} + 2\hat{S}^z_{n,m}\hat{S}^z_{n+1,m+1}) - \frac{A}{S^2}(\hat{S}^z_{n,m})^2\Bigg]. \tag{2}$$

In Eq. (2), $H = \frac{\tilde{H}}{\hbar^2 S^2}$; $J = \tilde{J}$; $J_1 = \tilde{J}_1$; $J_2 = \tilde{J}_2$ and $A = \tilde{A}$. In most of the ordered magnetic systems including the ferromagnets, the spin value is large which reduces the quantum fluctuation and hence a semiclassical description of the dynamics of the system becomes meaningful in these cases. In order to investigate the spin dynamics of the $(2+1)$ dimensional ferromagnetic spin system in the semiclassical limit, we bosonize the Hamiltonian by using the HP representation [17] of spin operators given by $\hat{S}^+_{n,m} = (2S)^{1/2}\left[1 - \frac{a^\dagger_{n,m}a_{n,m}}{2S}\right]^{1/2} a_{n,m}$, $\hat{S}^-_{n,m} = (2S)^{1/2}a^\dagger_{n,m}\left[1 - \frac{a^\dagger_{n,m}a_{n,m}}{2S}\right]^{1/2}$ and $\hat{S}^z_{n,m} = [S - a^\dagger_{n,m}a_{n,m}]$. The bosonic operators $a_{n,m}$ and $a^\dagger_{n,m}$ satisfy the usual commutation relations, $[a_{n,m}, a^\dagger_{n,m}] = 1$, $[a_m, a_n] = [a^\dagger_m, a^\dagger_n] = 0$. In the low temperature limit, for large spins, the ground state expectation value of $a^\dagger_{n,m}a_{n,m}$ is small compared to 2S and therefore the HP transformation for spin operators can be expanded in a power series of the dimensionless parameter $\epsilon = \frac{1}{\sqrt{S}}$ as

$$\frac{\hat{S}^+_{n,m}}{S} = \sqrt{2}\left[1 - \frac{\epsilon^2}{4}a^\dagger_{n,m}a_{n,m} - O(\epsilon^4)\right]\epsilon a_{n,m}, \tag{3}$$

$$\frac{\hat{S}^-_{n,m}}{S} = \sqrt{2}\epsilon a^\dagger_{n,m}\left[1 - \frac{\epsilon^2}{4}a^\dagger_{n,m}a_{n,m} - O(\epsilon^4)\right]. \tag{4}$$

By using Eqs. (3) and (4) in Eq. (2), we obtain the Hamiltonian that is written as a power series in $\epsilon$ as

$$\begin{aligned} H = \sum_{n,m}\{&m_1 + \epsilon^2[m_2 a_{n,m}a^\dagger_{n,m} \\ &+ J(a_{n+1,m}a^\dagger_{n+1,m} - a_{n+1,m}a^\dagger_{n,m} - a_{n,m}a^\dagger_{n+1,m}) \\ &+ J_1(a_{n,m+1}a^\dagger_{n,m+1} - a_{n,m+1}a^\dagger_{n,m} - a_{n,m}a^\dagger_{n,m+1}) \\ &+ J_2(a_{n+1,m+1}a^\dagger_{n+1,m+1} - a_{n+1,m+1}a^\dagger_{n,m} - a_{n,m}a^\dagger_{n+1,m+1})] \\ &+ \frac{\epsilon^4}{4}[4Aa^2_{n,m}a^{\dagger 2}_{n,m} + J(-4a_{n,m}a_{n+1,m}a^\dagger_{n,m}a^\dagger_{n+1,m} \\ &+ a_{n,m}a_{n+1,m}a^{\dagger 2}_{n,m} + a^2_{n,m}a^\dagger_{n,m}a^\dagger_{n+1,m} + a^2_{n+1,m}a^\dagger_{n,m}a^\dagger_{n+1,m} \\ &+ a_{n,m}a_{n+1,m}a^{\dagger 2}_{n+1,m}) + J_1(-4a_{n,m}a_{n,m+1}a^\dagger_{n,m}a^\dagger_{n,m+1} \\ &+ a_{n,m}a_{n,m+1}a^{\dagger 2}_{n,m} + a^2_{n,m}a^\dagger_{n,m}a^\dagger_{n,m+1} + a^2_{n,m+1}a^\dagger_{n,m}a^\dagger_{n,m+1} \\ &+ a_{n,m}a_{n,m+1}a^{\dagger 2}_{n,m+1}) + J_2(-4a_{n,m}a_{n+1,m+1}a^\dagger_{n,m}a^\dagger_{n+1,m+1} \\ &+ a_{n,m}a_{n+1,m+1}a^{\dagger 2}_{n,m} + a^2_{n,m}a^\dagger_{n,m}a^\dagger_{n+1,m+1} \\ &+ a^2_{n+1,m+1}a^\dagger_{n,m}a^\dagger_{n+1,m+1} + a_{n,m}a_{n+1,m+1}a^{\dagger 2}_{n+1,m+1})]\}, \end{aligned} \tag{5}$$

where $m_1 = (A - J - J_1 - J_2)$ and $m_2 = (-2A + J + J_1 + J_2)$. Now the spin dynamics can be expressed in terms of the Heisenberg equation of motion for the boson operator by substituting Eq. (5) in the following equation of motion:

$$i\hbar\frac{\partial a_{n,m}}{\partial t} = [a_{n,m}, H] = F(a^\dagger_{n,m}, a_{n,m}, a^\dagger_{n+1,m}, a_{n+1,m}, a^\dagger_{n,m+1}, a_{n,m+1}, a^\dagger_{n+1,m+1}, a_{n+1,m+1}), \tag{6}$$

where $F$ is given in Appendix A.

In particular, we are concerned with nonlinear excitations of spins induced by nonlinearity in the magnon system, in which a cluster of spins makes a trip as compared with the rest of the spins. Hence, we introduce Glauber's coherent-state representation [18] for the bosonic operators, $a^\dagger_{n,m}|u\rangle = u^*_{n,m}|u\rangle$, $a_{n,m}|u\rangle = u_{n,m}|u\rangle$, $|u\rangle = \Pi_{n,m}|u_{n,m}\rangle$ with $\langle u|u\rangle = 1$, where $u_{n,m}$ is the coherent amplitude of the operator $a_{n,m}$ for the system in the state $|u\rangle$. Now, we write down the equation of motion using Eq. (6) for the average $\langle u|a_{n,m}|u\rangle$ as

$$\begin{aligned} i\frac{\partial u_{n,m}}{\partial t} = \epsilon^2\{&-2m_1u_{n,m} - J(u_{n+1,m} + u_{n-1,m}) - J_1(u_{n,m+1} + u_{n,m-1}) \\ &- J_2(u_{n+1,m+1} + u_{n-1,m-1})\} + \frac{\epsilon^4}{4}\{8A|u_{n,m}|^2u_{n,m} \\ &+ J[-4u_{n,m}(|u_{n+1,m}|^2 + |u_{n-1,m}|^2) + 2|u_{n,m}|^2u_{n+1,m} \\ &+ u^2_{n,m}u^*_{n+1,m} + |u_{n-1,m}|^2u_{n-1,m} + |u_{n+1,m}|^2u_{n+1,m} \\ &+ u^2_{n,m}u^*_{n-1,m} + 2|u_{n,m}|^2u_{n-1,m}] + J_1[-4u_{n,m}(|u_{n,m+1}|^2 \\ &+ |u_{n,m-1}|^2) + 2|u_{n,m}|^2u_{n,m+1} + u^2_{n,m}u^*_{n,m+1} + |u_{n,m-1}|^2u_{n,m-1} \\ &+ |u_{n,m+1}|^2u_{n,m+1} + u^2_{n,m}u^*_{n,m-1} + 2|u_{n,m}|^2u_{n,m-1}] \\ &+ J_2[-4u_{n,m}(|u_{n+1,m+1}|^2 + |u_{n-1,m-1}|^2) + 2|u_{n,m}|^2u_{n+1,m+1} \\ &+ u^2_{n,m}u^*_{n+1,m+1} + |u_{n-1,m-1}|^2u_{n-1,m-1} + |u_{n+1,m+1}|^2u_{n+1,m+1} \\ &+ u^2_{n,m}u^*_{n-1,m-1} + 2|u_{n,m}|^2u_{n-1,m-1}]\}. \end{aligned} \tag{7}$$

Eq. (7) describes the nonlinear spin dynamics of an anisotropic ferromagnet in the discrete form.

## 3. Linear stability analysis

Various nonlinear dispersive wave systems exhibit instability known as the modulational instability (MI). The modulational instability is a fundamental phenomenon in nonlinear dispersive systems and is closely associated with the concept of self-localized waves or solitons. One of the main effects of modulational instability is the creation of localized pulses so that modulational