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Non-equilibrium energy transport in a thin metallic film: Analytical solution for radiative transport equation

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ABSTRACT

Non-equilibrium energy transfer in thin aluminum film is examined and analytical solution for the radiative transport equation is presented for the electron and lattice subsystems. Electron–phonon coupling is incorporated in the radiative transport equation to account for the energy exchange between the electron and the lattice sub-systems during their thermal communications. The radiative transport equations are reduced to a system of integral equations in the form of Fredholm integral equation of the second kind and they are solved analytically through the integral transformations technique. The analytical solution for temperature distribution in lattice and electron sub-systems is simulated for various film thicknesses. The analytical solution of two-equation model is also presented to compare the findings of the radiative transport in the film. The analytical solution of the radiative transport equation is validated through the numerical predictions. It is found that numerical predictions agree well with the findings of the analytical solution. Lattice site and electron temperatures obtained from the analytical solution differ from those resulted from the two-equation model for the aluminum film thickness of 0.1 μm because of the ballistic behavior of the phonons emitted at high temperature edge of the film.

1. Introduction

In thin metallic films, thermal separation takes place between electron and lattice sub-systems during the non-equilibrium energy transport. The non-equilibrium transport takes over the diffusional process because of the small film thickness, which is comparable to the mean free path of phonons. Thermal communication between the electron and the lattice sub-systems, which is governed by the electron–phonon coupling, forms the bases for the thermal energy transport in the film. Several approaches are introduced to formulate the non-equilibrium energy transport in the metallic films. Some of these approaches include hyperbolic telegraph equations [1], two-equation model [2], and electron kinetic model [3]. The previous approaches [1–3] provide solution to the heating problem, the findings do not give physical insight into the heating process including contribution of the ballistic phonons to the energy transport. However, the solution of Boltzmann transport equation provides information on the phonon intensity distribution during the thermal transport. Although numerical solution of the Boltzmann equation pertinent to non-equilibrium energy transport in thin films is possible [4], the analytical solution to the Boltzmann equation gives information between the heating parameters and the film characteristics. In addition, analytical solution minimizes the computational effort for the numerical solution of the heating problem. Consequently, investigation into the analytical solution of the Boltzmann equation becomes essential.

Considerable research studies were carried out to examine the solution of the Boltzmann equation. A multi-scale thermal device modeling incorporating the diffusion in the Boltzmann Transport Equation was carried out by Pisipati et al. [5]. The simulation of the thermal model was performed using the COMSOL multi-physics finite element code. In addition, they discussed the Boltzmann transport model with and without diffusion for multilayers including the interface conditions. Coupling of heat and momentum transfer between nanostructured surfaces was investigated by Donkov et al. [6]. They derived the expression for the thermally-induced force as a function of the geometric parameters characterizing the surface topography and compare the results with the findings of the Monte-Carlo simulations. In addition, they indicated that when the surfaces were held at different temperatures the heat transfer was accompanied by a transfer of momentum such that a force was created parallel to the surface. Thermal conduction in the anisotropy of silicon nano-films was studied by Terris et al. [7]. They showed that the cross-plane thermal conduction appeared to be less than the in-plane thermal conduction due to the presence of anisotropy. Longitudinal thermal conductivity of radially hetero-structured nano-wire was examined by Lu [8]. He demonstrated that the thermal conductivity of the structures could be modulated by

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Nomenclature

C	volumetric specific heat capacity of the dielectric material
C_e	volumetric specific heat capacity of the electron sub-system
C_p	volumetric specific heat capacity of the phonon sub-system
G	electron–phonon coupling constant
I	frequency independent phonon intensity
I^+	forward intensity
I^{-1}	backward intensity
I^*	equilibrium phonon intensity
k_e	electron sub-system thermal conductivity
k_p	phonon sub-system thermal conductivity
L	film thickness
t	time
T	temperature
T_e	electron sub-system temperature
T_p	phonon sub-system temperature
v	speed of sound
v_p	speed of sound in lattice sub-system
v_e	speed of sound in electron sub-system
x	Cartesian coordinate

Greek symbols

γ	$\frac{G}{C\upsilon}$
λ	$\frac{1}{\Lambda}-\frac{G}{C\upsilon}$
Λ	phonon mean free path
μ	cosine of the azimuthal angle
φ	azimuthal angle

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e	electron
p	phonon

changing the radius ratio between the shell layer and the core layer of the nanowire heterostructures and the findings might serve as a possible way for tuning the thermal conductivity of nanostructures. The size-dependent thermal conductivity in nano-systems based on non-Fourier heat transfer was investigated by Ma [9]. He presented the analytical model to predict the effective thermal conductivity along dielectric thin films or nanowires with smooth wall surfaces. The size effect in micro and nano-structures was studied by Wenjun et al. [10]. The model developed could be conveniently used to approximate the solution of the Boltzmann transport equation and the results could be used for the predictions of the electrical resistance and/or thermal conductivity of the metallic substrates as well as semiconductor nano-structures. Phonon heat conduction in micro/nano-cylindrical and spherical media was investigated by Zeng [11]. He showed that the phonon transmission and reflection were the dominant factor for determining the equivalent thermal conductivity of micro/nano-cylindrical and spherical media and significant temperature drop occurred at the interface.

Although numerical solution of the Boltzmann equation is possible, the simulations require excessive computer power and computational efforts. Analytical solution to the Boltzmann equation reduces the computational efforts and provides the analytical relations between the film properties and the phonon intensity distribution. However, the analytical solutions presented in the previous study [12] is limited with the emitted and the reflected phonon intensities across the film and the solution for the equilibrium intensity is not provided. Therefore, in the present study, analytical solution of the Boltzmann equation pertinent to metallic thin film is presented. Since the thermal separation takes place between the electron and the lattice sub-system, the solution covers the radiative energy transfer in both sub-systems. The results are extended to include the effects of aluminum film thickness on the phonon intensity distribution.

2. Mathematical analysis

The Boltzmann transport equation can be reduced to a phonon radiative transport equation, which can be written as [13]:

$$\frac{1}{v}\frac{\partial I}{\partial t}+\mu\frac{\partial I}{\partial x}=\frac{\frac{1}{2}\int_{-1}^{1} I d\mu - I}{\Lambda} \tag{1}$$

In Eq. (1), $I=I\,(x,\mu,t)$ is the phonon intensity in units of W/m^2, x is the distance along the film thickness, $\mu=\cos(\theta)$, θ is the angle of the velocity (momentum) vector of a phonon from the x-axis and t is the time. v is the phonon speed and Λ is the phonon mean free path. However, for the metal thin films, thermal separation of electron and phonon sub-systems takes place during the heating process and re-structuring of the energy equation becomes necessary to account for the energy transport in each sub-system. Therefore, in each sub-system, a separate equation for radiative phonon transport should be written in line with the Boltzmann equation. The thermal communication between the electron and the lattice sub-systems takes place through electron–phonon coupling process. In order to account for the thermal communication of the sub-systems, electron–phonon coupling needs to be incorporated in the governing equation of radiative phonon transport.

2.1. The coupled system of equations of phonon radiative transfer in lattice sub-system of aluminum film

Energy transport in the aluminum thin films is considered to be governed by the radiative transport. In order to analyze the energy transport in the lattice sub-system, the modified Boltzmann equation satisfying the conservation of energy can be written as:

$$\frac{1}{v_p}\frac{\partial I_p}{\partial t}+\mu\frac{\partial I_p}{\partial x}=\frac{\frac{1}{2}\int_{-1}^{1} I_p d\mu - I_p}{\Lambda_p}-\frac{G}{2}\left(\frac{1}{C_p v_p}\int_{-1}^{1} I_p d\mu-\frac{1}{C_e v_e}\int_{-1}^{1} I_e d\mu\right) \tag{2}$$

In the case of the electron sub-system, the proposed equation for the energy transport satisfying the conservation of energy can be written as:

$$\frac{1}{v_e}\frac{\partial I_e}{\partial t}+\mu\frac{\partial I_e}{\partial x}=\frac{\frac{1}{2}\int_{-1}^{1} I_e d\mu - I_e}{\Lambda_e}-\frac{G}{2}\left(\frac{1}{C_e v_e}\int_{-1}^{1} I_e d\mu-\frac{1}{C_p v_p}\int_{-1}^{1} I_p d\mu\right) \tag{3}$$

In Eqs. (2) and (3) above, I_p, v_p, Λ_p and $C_{p,}$ G are the phonon intensity, speed, mean free path, and volumetric specific heat and the electron–phonon coupling constant, respectively.

In the case of the steady transport, Eq. (2) satisfies the conservation of energy and it reduces to following equations for emitted and reflected phonon intensities (Fig. 1).