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ORIGINAL ARTICLE

Analytical study for singular system of transistor circuits

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Abstract In this paper, we propose a user friendly algorithm based on homotopy analysis transform method for solving observer design in generalized state space or singular system of transistor circuits. The homotopy analysis transform method is an innovative adjustment in Laplace transform method and makes the calculation much simpler. The effectiveness of technique is described and illustrated with an example. The obtained results are in a good agreement with the existing ones in open literature and it is shown that the scheme proposed here is robust, efficient, easy to implement and computationally very attractive.

1. Introduction

The homotopy analysis method (HAM) was first proposed and applied by Liao [1–5] based on homotopy, a fundamental concept in topology and differential geometry. The HAM has been successfully applied by many researchers for solving linear and non-linear partial differential equations [6–12]. In recent years, many authors have paid attention to study the solutions of the linear and nonlinear partial differential equations by using various methods combined with the Laplace transform method. Among these are Laplace decomposition method (LDM) [13–15], homotopy perturbation transform method (HPTM) [16–18] and homotopy analysis transform method (HATM) [19–25].

In this paper, we present a simplest model for a transistor in the circuit. Previously such type of model has been studied by Kang [26] and Lewis [27]. Furthermore, Balachandran and Murugesan [28,29] and Kalpana and Raja Balachandar [30] have applied the Single Term Walsh Series (STWS) technique and Haar Wavelet method respectively to solve the model for a transistor presented in [26,27]. Very recently Krishnaveni et al. [31] and Singh et al. [32] applied the Adomian's decomposition method (ADM) and Laplace decomposition method (LDM) respectively to solve singular system of transistor circuits. In this article, we implement the homotopy analysis transform method (HATM) to solve singular system of transistor circuits. The HATM is an elegant combination of the Laplace trans-

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form method and HAM. The advantage of this technique is its capability of combining two powerful methods for obtaining exact and approximate analytical solutions for nonlinear equations. It is worth mentioning that the proposed method is capable of reducing the volume of the computational work as compared to the classical methods while still maintaining the high accuracy of the numerical result; the size reduction amounts to an improvement of the performance of the approach.

2. Analysis of singular systems by homotopy analysis transform method (HATM)

Consider the linear singular system of the form

$$K\dot{X}(t) = AX(t) + BU(t), \tag{1}$$

$$Y(t) = CX(t), \quad X(0) = X_0, \tag{2}$$

with $X \in R^n$, $U \in R^m$, $Y \in R^p$, K generally a singular matrix. $Y(t)$ is a 'p' output vector. A, B and C are to be chosen as constant matrices with appropriate dimensions. $X(t)$ is an 'n' state vector, $U(t)$ is an 'm' input vector. Now, we discuss the HATM to solve (1). Applying the Laplace transform on both sides of Eq. (1) and using (2), we get

$$KL[X(t)] - \frac{K}{s}X_0 - \frac{A}{s}L[X(t)] - \frac{B}{s}L[U(t)] = 0. \tag{3}$$

We define the nonlinear operator as

$$N[\phi(t;q)] = KL[\phi(t;q)] - \frac{K}{s}X_0 - \frac{A}{s}[L\ [\phi(t;q)]] - \frac{B}{s}L[U(t)], \tag{4}$$

where $q \in [0, 1]$ and $\phi(t; q)$ is a real function of t and q. We construct a homotopy as follows

$$(1-q)L[\phi(t;q) - X_0(t)] = \hbar q H(t) N[X(t)], \tag{5}$$

where L denotes the Laplace transform, $q \in [0, 1]$ is the embedding parameter, $H(t)$ denotes a nonzero auxiliary function, $\hbar \neq 0$ is an auxiliary parameter, $X_0(t)$ is an initial guess of $X(t)$ and $\phi(t; q)$ is a unknown function. Obviously, when the embedding parameter $q = 0$ and $q = 1$, it holds

$$\phi(t;0) = X_0(t), \qquad \phi(t;1) = X(t), \tag{6}$$

respectively. Thus, as q increases from 0 to1, the solution $\phi(t; q)$ varies from the initial guess $X_0(t)$ to the solution $X(t)$. Expanding $\phi(t; q)$ in Taylor series with respect to q, we have

$$\phi(t;q) = X_0(t) + \sum_{m=1}^{\infty} X_m(t) q^m, \tag{7}$$

where

$$X_m(t) = \frac{1}{m!}\frac{\partial^m \phi(t;q)}{\partial q^m}\bigg|_{q=0}. \tag{8}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter $\hbar$, and the auxiliary function are properly chosen, the series (7) converges at $q = 1$, then we have

$$X(t) = X_0(t) + \sum_{m=1}^{\infty} X_m(t), \tag{9}$$

which must be one of the solutions of the original equations. According to the definition (9), the governing equation can be deduced from the zero-order deformation (5). Define the vectors

$$\vec{X}_m = \{X_0(t), X_1(t), \ldots, X_m(t)\}. \tag{10}$$

Differentiating the zeroth-order deformation Eq. (5) m-times with respect to q and then dividing them by $m!$ and finally setting $q = 0$, we get the following mth-order deformation equation:

$$L[X_m(t) - \chi_m X_{m-1}(t)] = \hbar q H(t) \Re_m(\vec{X}_{m-1}). \tag{11}$$

Applying the inverse Laplace transform, we have

$$X_m(t) = \chi_m X_{m-1}(t) + \hbar L^{-1}[qH(t)\Re_m(\vec{X}_{m-1})], \tag{12}$$

where

$$\Re_m(\vec{X}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\phi(t;q)]}{\partial q^{m-1}}\bigg| q = 0, \tag{13}$$

and

$$\chi_m = \begin{cases} 0, & m \leqslant 1, \\ 1, & m > 1. \end{cases} \tag{14}$$

3. Analysis of transistor circuits

In this section, we discuss the implementation of our proposed method and investigate its accuracy by applying the HAM with coupling of the Laplace transform method. Consider the simplest model for a transistor in the circuit shown in Fig. 1 investigated by Kang [26] and Lewis [27]. The circuit equations are given by

$$u_1 + x_1 + r_1 x_2 = 0,$$

$$u_2 + x_3 + r_2(x_4 - \alpha_1 x_2) = 0,$$

$$x_2 = c_1 \dot{x}_1,$$

$$x_4 = c_2 \dot{x}_3,$$

$$y_1 = r_2(\alpha_1 x_2 - x_4),$$

$$y_2 = r_L \alpha_2 x_4, \tag{15}$$

where x_1, x_2, x_3 and x_4 are the capacity voltages. Assuming $r_1 = r_2 = r_L = \alpha_1 = \alpha_2 = c_1 = c_2 = 1$, we get a singular system

$$K\dot{X}(t) = AX + BU,$$

$$Y = CX, \tag{16}$$

where

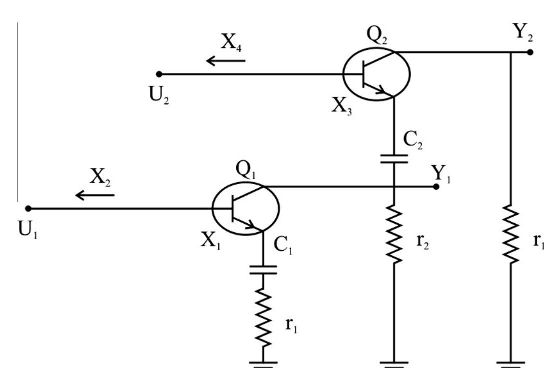

Figure 1 Model for a transistor circuit.