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ORIGINAL ARTICLE

Exact solution of peristaltic flow of biviscosity fluid in an endoscope: A note

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Abstract This article deals with the peristaltic flow of biviscosity fluid in an endoscope. This study concentrates on the MHD characteristics and endoscope effects for peristaltic flow of a biviscosity fluid. Exact solutions are computed. The importance of pertinent flow parameters entering into the flow modeling is discussed. It is observed that increases in viscosity coefficient β, Hartmann number M and value of radius ratio ε cause increase in all the pumping regions.

1. Introduction

The peristaltic flow problems in a tube, annulus, endoscope, and channels have been discussed by various researchers [1–14] because of its applications in physiology and industry. Hakeem et al. [15] have discussed the hydromagnetic flow of generalized Newtonian fluid through a uniform tube with peristalsis. The peristaltic flow of MHD Newtonian fluid in an endoscope was studied by Mekheimer and Elmaboud [16]. Hariharan et al. [17] have investigated the peristaltic transport of non-Newtonian fluid in a diverging tube with different wave forms. The peristaltic flow of a third order fluid in an endoscope has been analyzed by Nadeem et al. [18]. Mekheimer and elmaboud discussed the peristaltic flow of a couple stress fluid in an annulus: Application of an endoscope [19] has discussed the peristaltic transport of a third order fluid in a cylindrical tube. The peristaltic flow and heat transfer in a vertical annulus with long wave approximation have been studied by Vajravelu et al. [20]. Some more recent article related to the topic is Refs. [21–24].

In view of the above analysis the purpose of present investigation is to discuss the peristaltic flow of a biviscosity fluid model in an endoscope under the assumptions of long wave length and low Reynolds number. Exact solutions are computed. The importance of pertinent flow parameters entering into the flow modeling is discussed.

2. Formulation of the problem

Consider the MHD flow of an incompressible electrically conducting biviscosity fluid in a uniform endoscope of varying cross-section with a sinusoidal wave of small amplitude travelling down its wall. The geometry of the wall surface is defined as

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$$\overline{R}_1 = a_1, \tag{1}$$

$$\overline{R}_2 = a_2 + b\sin\frac{2\pi}{\lambda}(\overline{Z} - c\bar{t}). \tag{2}$$

where a_1 is the radius of the inner tube, a_2 is the radius of the outer tube at inlet, b is the wave amplitude, λ is the wavelength, c the wave speed, $\bar{t}$ the time and b is the wave amplitude.

In the fixed frame $(\overline{R}, \overline{Z})$ the continuity, momentum and energy equations give

$$\frac{\partial \overline{V}_r}{\partial \overline{R}} + \frac{\overline{V}_r}{\overline{R}} + \frac{\partial \overline{V}_z}{\partial \overline{Z}} = 0, \tag{3}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \overline{V}_r\frac{\partial}{\partial \overline{R}} + \overline{V}_z\frac{\partial}{\partial \overline{Z}}\right)\overline{V}_r = -\frac{\partial \bar{p}}{\partial \overline{R}} + \frac{1}{\overline{R}}\frac{\partial}{\partial \overline{R}}(\overline{R}\overline{S}_{\overline{RR}}) + \frac{\partial}{\partial \overline{Z}} \times (\overline{S}_{\overline{RZ}}) - \frac{\overline{S}_{\bar{\theta}\bar{\theta}}}{\overline{R}}, \tag{4}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \overline{V}_r\frac{\partial}{\partial \overline{R}} + \overline{V}_z\frac{\partial}{\partial \overline{Z}}\right)\overline{V}_z = -\frac{\partial \bar{p}}{\partial \overline{Z}} + \frac{1}{\overline{R}}\frac{\partial}{\partial \overline{R}}(\overline{R}\overline{S}_{\overline{RZ}}) + \frac{\partial}{\partial \overline{Z}} \times (\overline{S}_{\overline{ZZ}}), \tag{5}$$

where $\bar{p}$ is the pressure and $\overline{V}_r$, $\overline{V}_z$ are the respective velocity components in the radial and axial directions respectively and ρ is the density.

The coordinates in the fixed $(\overline{R}, \overline{Z})$ and wave $(\bar{r}, \bar{z})$ frames are related through the following transformations

$$\bar{r} = \overline{R}, \quad \bar{z} = \overline{Z} - c\bar{t}, \quad \bar{v}_r = \overline{V}_r, \quad \bar{v}_z = \overline{V}_z - c. \tag{6}$$

The constitutive equation for incompressible biviscosity fluids [4] is defined as follow

$$S_{ij} = \begin{cases} 2(\mu_\beta + p_y/\sqrt{2\pi})e_{ij}, & \pi \succeq \pi_c, \\ 2(\mu_\beta + p_y/\sqrt{2\pi_c})e_{ij}, & \pi < \pi_c, \end{cases} \tag{7}$$

We introduce the following non-dimensional parameter $\beta = \mu_\beta\sqrt{2\pi_c}/p_y$, where p_y is yield stress and $\pi = e_{ij}$, e_{ij} is the (i,j) component of deformation rate, β denotes the upper limit apparent viscosity coefficient, and μ_β is the plastic viscosity of the fluid. For ordinary fluid $(p_y = 0)$, we have

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial yi}\right). \tag{8}$$

Here we use the following dimensionless quantities

$$r = \frac{\bar{r}}{a_2}, \quad z = \frac{\bar{z}}{\lambda}, \quad v_z = \frac{\bar{v}_z}{c}, \quad v_r = \frac{\lambda\bar{v}_r}{a_2 c}, \quad P = \frac{a_2^2\bar{p}}{c\lambda\mu},$$

$$t = \frac{c\bar{t}}{\lambda}, \quad \delta = \frac{a_2}{\lambda}, \quad Re = \frac{\rho c a_2}{\mu}, \quad r_1 = \frac{\bar{r}_1}{a_2} = \varepsilon$$

$$r_2 = \frac{\bar{r}_2}{a_2} = 1 + \phi\sin(2\pi z), \quad S = \frac{a_2\overline{S}}{c\mu}. \tag{9}$$

Using Eqs. (6)–(9) in Eqs. (3)–(5) we have the following governing equations in dimensionless can be written as [4]

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial r} = 0, \tag{10}$$

$$Re\delta^3\left(v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + (1+\beta^{-1})\,\delta^2\left[2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{\partial^2 v_z}{\partial r\partial z} + \delta^2\frac{\partial^2 v_r}{\partial z^2}\right)\right]. \tag{11}$$

$$Re\delta\left(v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + (1+\beta^{-1})\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \delta^2\left(\frac{\partial v_r}{\partial r} + 2\frac{\partial v_z}{\partial z} + \frac{v_r}{r}\right)\right] - M^2(v_z+1). \tag{12}$$

After applying the long wavelength approximation $(\delta = 0)$ and neglecting the wave number along with low Reynolds number we get r, z component of the momentum of the form

$$\frac{\partial p}{\partial r} = 0, \tag{13}$$

$$(1+\beta^{-1})\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right) - M^2(v_z+1) = \frac{\partial p}{\partial z}. \tag{14}$$

The appropriate boundary condition

$$v_z = -1 \text{ at } r = r_1, \tag{14a}$$

$$v_z = -1 \text{ at } r = r_2 = 1 + \phi\sin(2\pi z). \tag{14b}$$

3. Solution of the problem

3.1. Exact solution

The governing Eqs. (13) and (14) with the help of Eqs. (14a) and (14b) give

$$v_z = -1 + \alpha\frac{dp}{dz}\frac{1}{M_1^2}\left(\frac{a_{13}I_0(M_1 r) + a_{12}K_0(M_1 r)}{a_{11}} - 1\right). \tag{15}$$

The instantaneous volume flow rate in the fixed coordinate system is given by [19]

$$Q = F + \frac{1}{2}\left(1 - \varepsilon^2 + \frac{\phi^2}{2}\right). \tag{16}$$

The volume flow rate F in the moving frame is given by,

$$F = \int_{r_1}^{r_2} r v_z dr. \tag{17}$$

Substitution of Eq. (15) in Eq. (17) and then solve the result for dp/dz we have

$$\frac{dp}{dz} = \frac{1}{a_{14}}\left(Q + \varepsilon^2 - \left(1 + \frac{\phi^2}{2}\right) - a_{15}\right). \tag{18}$$

where

$$M_1 = \frac{M^2}{(1+\beta^{-1})}, \quad \alpha = \frac{1}{(1+\beta^{-1})},$$

$$a_{11} = K_0(M_1 r_1)I_0(M_1 r_2) - K_0(M_1 r_2)I_0(M_1 r_1),$$

$$a_{12} = I_0(M_1 r_2) - I_0(M_1 r_1), \quad a_{13} = K_0(M_1 r_1) - K_0(M_1 r_2),$$

$$a_{14} = 2\alpha\frac{1}{M_1^2}\left(\frac{a_{13}}{M_1 a_{11}}(r_2 I_1(M_1 r_2) - r_1 I_1(M_1 r_1)) - \frac{a_{12}}{a_{11}M_1}(r_2 K_1(M_1 r_2) - r_1 K_1(M_1 r_1)) - \frac{(r_2^2 - r_1^2)}{2}\right),$$