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ORIGINAL ARTICLE

Unsteady stagnation point flow of second grade fluid with variable free stream

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Abstract This article discusses the stagnation-point flow of second grade fluid over an unsteady stretching surface in the presence of variable free stream. Flow analysis has been carried out with heat transfer analysis. The resulting partial differential equations have been converted into ordinary differential equations by employing the suitable transformations. Computations of dimensionless velocity and temperature fields have been performed by using homotopy analysis method (HAM). Graphs are plotted to examine the behaviors of arising physical parameters on the dimensionless velocity and temperature. Numerical values of skin-friction coefficient and local Nusselt number are computed and examined.

1. Introduction

The range of non-Newtonian fluids is very large because of their occurrence in the engineering and industrial processes. Non-Newtonian fluids have been investigated by the several researchers under various conditions (see [1–10]). Such fluids are specifically quite common in the process of manufacturing coated sheets, foods, optical fibers, drilling muds, plastic polymers, etc. It is well known that all the non-Newtonian fluids cannot be described by a single constitutive relationship in view of their diverse characteristics. Hence several models of non-Newtonian fluids have been suggested. The governing equations in the non-Newtonian fluids in general are much complicated, more nonlinear and higher order than the Navier–Stokes equations. The non-Newtonian fluids have been mainly classified into three types which are called the differential, the rate and the integral. Out of these, the differential type fluids have been attracted much by the researchers. A simplest subclass of differential type model is called a second grade fluid which we aim to study here.

The stagnation point flow and heat transfer over a stretching sheet is further important in the process of polymer extrusion, paper production, insulating materials, glass drawing, continuos casting, fine fiber matts and many others. Considerable progress has been made regarding the stretching and stagnation point flows in the past. Chiam [11] studied the two-dimensional stagnation-point flow of a viscous fluid

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toward a linear stretching surface. Mahapatra and Gupta [12] analyzed the effects of heat transfer in the stagnation point flow toward a stretching surface. The steady stagnation point flow of an incompressible micropolar fluid over a stretching surface is studied by Nazar et al. [13]. Sadeghy et al. [14] numerically studied the stagnation point flow of an upper convected Maxwell fluid. Ishak et al. [15] investigated the mixed convection stagnation point flow of an incompressible viscous fluid toward a vertical permeable stretching sheet. The effect of thermal radiation on the mixed convection boundary layer magnetohydrodynamic stagnation point flow in a porous space was investigated by Hayat et al. [16]. Mahapatra et al. [17] discussed the steady two-dimensional oblique stagnation point flow of an incompressible viscoelastic fluid toward a stretching sheet. Labropulu and Li [18] examined the steady two dimensional stagnation point flow in the presence of slip effects. Literature survey depicts that much attention has been given to the stretching flows in steady situations. There are very few attempts which shed light on the time-dependent stretching flow problems [19–24]. Sharma and Singh [25] numerically studied the flow about a stagnation point over an unsteady stretching sheet in the presence of variable free stream.

The purpose of present study is to investigate the stagnation point flow of a second grade fluid in the presence of variable free stream. The paper is structured as follows. The problem is formulated in section two. Sections three and four deal with the solutions by the homotopy analysis technique [26–33] and their convergence respectively. Results and discussion are given in section five. Section six consists of concluding remarks.

2. Mathematical formulation

Consider the unsteady stagnation point flow of an incompressible second grade fluid over a porous stretching surface with variable free stream. We select x-axis along the surface and y-axis normal to it. In addition, the heat transfer is considered. The boundary layer equations which can govern the present flow problem are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+v\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left[\begin{array}{c}\frac{\partial^3 u}{\partial t\partial y^2}+u\frac{\partial^3 u}{\partial x\partial y^2}\\ +\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\end{array}\right] \tag{2}$$

$$\rho c_p\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v being the velocity components along the x- and y-axes, U the free stream velocity, α_1 the second grade parameter, ρ the fluid density, v the kinematic viscosity, T the fluid temperature, ρ the fluid density and c_p the specific heat.

The associated boundary conditions of the present flow analysis are (see Fig. 1):

$$u=U_w(x,t),\ \ v=V_w(x,t),\ \ T=T_w(x,t)\ \text{at}\ y=0,$$
$$u\to U(x,t)=,\ \ T\to T_\infty\ \text{as}\ y\to\infty. \tag{4}$$

with V_w defined

$$V_w=-\frac{v_0}{(1-\varepsilon t)^{1/2}}, \tag{5}$$

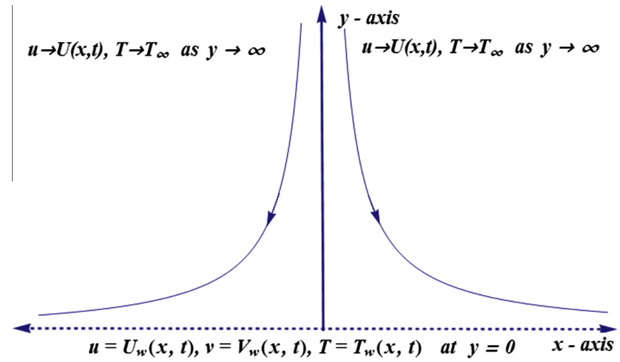

Fig. 1 Physical model.

where V_w represents the mass transfer at surface with $V_w>0$ for injection and $V_w<0$ for suction and v_0 is a constant measuring the strength of applied suction/injection. Further the stretching velocity $U_w(x,t)$ and surface temperature $T_w(x,t)$ are taken as follows

$$U_w(x,t)=\frac{cx}{1-\varepsilon t},$$
$$T_w(x,t)=T_\infty+\frac{bx}{1-\varepsilon t},$$
$$U(x,t)=\frac{ax}{1-\varepsilon t} \tag{6}$$

in which a, c and ε are the constants with $a>0$ and $\varepsilon\geqslant 0$ (with $\varepsilon t<1$) and both a and ε have dimension time^{-1}.

We introduce the following transformations [23]:

$$\eta=\sqrt{\frac{U_w}{vx}}y,\ \ \psi=\sqrt{vxU_w}f(\eta),\ \ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{7}$$

and the velocity components

$$u=\frac{\partial\psi}{\partial y}\ v=-\frac{\partial\psi}{\partial x}, \tag{8}$$

where ψ is a stream function. The continuity equation is identically satisfied and the resulting problems for f and θ long with the boundary conditions are

$$f'''-f'^2+ff''-A\left(f'+\frac{1}{2}\eta f''\right)$$
$$+\alpha\left[2f'f'''-f''^2-ff''''+A\left(2f'''+\frac{1}{2}\eta f''''\right)\right]+\frac{a^2}{c^2}+A\frac{a}{c}=0, \tag{9}$$

$$\theta''+\Pr(f\theta'-f'\theta)-\Pr A\left(\theta+\frac{1}{2}\eta\theta'\right), \tag{10}$$

$$f(0)=S,\ \ f'(0)=1,\ \ f'(\infty)\to a/c,\ \ \theta(0)=1,\ \theta(\infty)\to 0. \tag{11}$$

Here $A=\varepsilon/c$ is the unsteadiness parameter, $\alpha=c\alpha_1/\mu(1-\varepsilon t)$ the dimensionless second grade parameter, $\Pr=\frac{\mu c_p}{k}$ the Prandtl number and primes indicate the differentiation with respect to η.

The skin friction coefficient C_f and local Nusselt number Nu_x are defined below

$$C_f=\frac{\tau_w}{\rho u_w^2},$$
$$Nu=\frac{xq_w}{k(T_w-T_\infty)}, \tag{12}$$

where the skin-friction τ_w and wall heat flux q_w are defined as [33]: