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ORIGINAL ARTICLE

On unsteady two-dimensional and axisymmetric squeezing flow between parallel plates

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Abstract Squeezing flow of a viscous fluid is considered. Two types of flows are discussed namely, the axisymmetric flow and two dimensional flow. Similarity transform proposed by Wang (1976) [13] has been used to reduce the Navier–Stokes equations to a highly non-linear ordinary differential equation which jointly describes both types of flows. Solution to aforementioned ordinary differential equation is obtained by using Variation of Parameters Method (VPM). VPM is free from the existence of small or large parameters and hence it can be applied to a large variety of problems as compared to the perturbation method applied by Wang (1976) [13]. Comparison among present and already existing solutions is also provided to show the efficiency of VPM. A convergence analysis is also carried out. Effects of different physical parameters on the flow field is discussed and demonstrated graphically with comprehensive discussions and explanations.

1. Introduction

Squeezing flow between parallel walls accrues in many industrial and biological systems. Moving pistons in engines, hydraulic brakes, chocolate filler and many other devices are based on the principle of flow between contracting domains. To develop these equipment and machines better understanding of such flow models which describe the squeezing flow between parallel walls is always needed. Classical work in this regard can be traced back to Stefan [1], who presented his work on squeezing flow by using lubrication assumption. Later in 1986 Reynolds [2] studied the case for elliptic plates, and Archibald [3] considered the squeezing flow between rectangular plates. After that several researchers have contributed their efforts to make squeezing flow model more understandable [4–8].

Earlier studies on squeezing flows are based on Reynolds equation however the scantiness of Reynolds equation for some cases has been shown by Jackson [9]. More flexible and useful similarity transforms are now available due to the efforts of Birkhoff [10], Yang [11] and Wang and Watson

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[12]. These similarity transforms reduce the Navier–Stokes equation into a fourth order nonlinear ordinary differential equation and have further been used in some other investigations as well [13–17].

Most of the real world problems are inherently in the form of nonlinearities. Over the years much attention has been devoted to develop new efficient analytical techniques that can cope up with such nonlinearities. Several approximation techniques have been developed to fulfill this purpose [18–27]. Nowadays, researchers prefer those techniques which are easy to implement, require less computational work and time to provide reliable results. One of these analytical techniques is Variation of Parameters Method (VPM) [28,29]. Main advantages of VPM are that it does not depend on existence of small or large parameters; it is free from round off errors, calculation of so called Adomian's polynomials, linearization or discretization. It uses initial conditions that are easier to be implemented and reduces the computational work while still maintaining a higher level of accuracy. One can easily access the recent applications of VPM in different available studies [30–33].

In this study one may clearly see that VPM can successfully be applied to solve the equations governing unsteady squeezing flows between parallel plates. Comparison of the results obtained by VPM to the numerical solution obtained by using Runge–Kutta order 4 is also provided to show the effectiveness of the technique. Obtained results are also compared with already existing studies. A convergence analysis is also carried out to check the computational cost benefits of VPM. It is evident from this article that VPM provides better results with less amount of laborious computational work.

2. Governing equations

Consider an incompressible flow of a viscous fluid between two parallel plates separated by a distance $z = \pm l(1-\alpha t)^{1/2} = \pm h(t)$, where l is the position at time $t = 0$. For $\alpha > 0$ plates are squeezed until they touch each other at $t = 1/\alpha$ for $\alpha < 0$ plates are separated. Let u, v and w be the velocity components in x, y and z directions respectively, shown in Fig. 1. Using transform introduced by Wang [13] for a two-dimensional flow:

$$u = \frac{\alpha x}{[2(1-\alpha t)]} F'(\eta), \tag{1}$$

$$w = \frac{-\alpha l}{[2(1-\alpha t)^{1/2}]} F(\eta), \tag{2}$$

where,

$$\eta = \frac{z}{\left[l(1-\alpha t)^{1/2}\right]}. \tag{3}$$

Substituting, Eqs. (1)–(3) in unsteady two-dimensional Navier–Stokes equations yield a non-linear ordinary differential equation of same form as discussed by [17],

$$F^{iv}(\eta) + S(-\eta F(\eta) - 3F''(\eta) - F'(\eta)F''(\eta) + F(\eta)F'''(\eta)) = 0, \tag{4}$$

where $S = \alpha l^2/2\nu$ is the non-dimensional Squeeze number, and ν is the kinematic viscosity. Boundary conditions for the problem are such that on plates the lateral velocities are zero and normal velocity is equal to velocity of the plate, that is

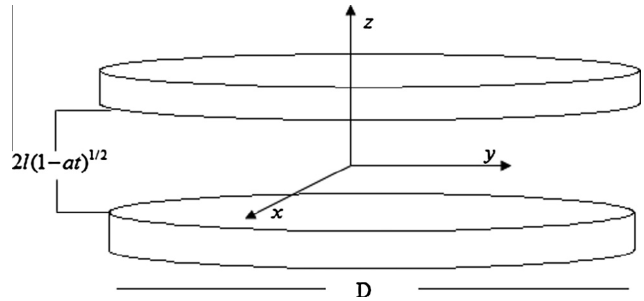

Figure 1 Schematic diagram of the problem.

$$F(0) = 0, \quad F''(0) = 0, \quad F(1) = 1, \quad F'(1) = 0. \tag{5}$$

Similarly for the axisymmetric case, transforms introduced by Wang [13] are

$$u = \frac{\alpha x}{[4(1-\alpha t)]} F'(\eta), \tag{6}$$

$$v = \frac{\alpha y}{[4(1-\alpha t)]} F'(\eta), \tag{7}$$

$$w = \frac{-\alpha l}{[2(1-\alpha t)]} F(\eta). \tag{8}$$

Using Eqs. (6)–(8) in unsteady axisymmetric Navier–Stokes equations we get a nonlinear ordinary differential equation of the form

$$F^{iv}(\eta) + S(-\eta F(\eta) - 3F''(\eta) + F(\eta)F'''(\eta)) = 0. \tag{9}$$

Thus, we have to solve non-linear ordinary differential equation of the form

$$F^{iv}(\eta) + S(-\eta F(\eta) - 3F''(\eta) - \beta F'(\eta)F''(\eta) + F(\eta)F'''(\eta)) = 0, \tag{10}$$

subject to the boundary conditions given in Eq. (5).

In Eq. (10), $\beta = 0$ corresponds to axisymmetric flow while $\beta = 1$ gives two-dimensional case.

3. Solution procedure

Following the standard procedure proposed for VPM [28–33], we can write Eq. (10) as

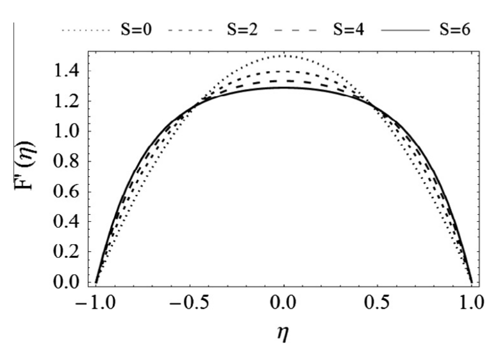

Figure 2 Effects of S on $F'(\eta)$ in expanding motion of plates (axisymmetric case).