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Dynamical response of ultracold interacting fermion–boson mixtures

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ABSTRACT

We analyze the dynamical response of a ultracold binary gas mixture in the presence of strong boson–fermion couplings. Mapping the problem onto that of the optical response of a metal/semiconductor electronic degrees of freedom to electromagnetic perturbation we calculate the corresponding dynamic linear response susceptibility in the non-perturbative regimes of strong boson–fermion coupling using diagrammatic resummation technique as well as quantum Monte Carlo simulations. We evaluate the Bragg spectral function as well as the optical conductivity and find a pseudogap, which forms in certain parameter regimes.

1. Introduction

Ultracold gas mixtures of bosons and fermions, which arise very naturally in the sympathetic cooling processes, are very interesting quantum systems with physical properties very different from conventional quantum gases [1,2]. In view of recent advances in the field of correlated ultracold gases it is very important to understand their dynamical properties, e.g. their response to an external dynamical scattering potential brought about by a change of trapping potential. If the bosonic subsystem is dominantly in the BEC phase the effective low-energy interaction with the fermionic subsystem is the coupling between the latter and the phonons (sound waves) of the former. From the mathematical point of view such a system is nothing but an electron–phonon coupled system best described by the Fröhlich Hamiltonian, see Eq. (1) in Section 2, the interaction term of which describes precisely the scattering of fermions on the bosonic degrees of freedom mentioned above [3]. In the case of low fermion concentrations it describes individual impurities imbedded into a continuum of massless bosonic modes. Under such conditions the physics of the system is supposed to be very close to that of the classical polaron, taking place in semiconductors with strong electron–phonon interaction [4–6].

There are, however, fundamental differences between the conventional (solid state) polarons and their BEC counterparts. The most obvious one is the different phonon spectrum of the bosonic subsystem as well as an explicit momentum dependence of the electron-phonon coupling [6]. While these details do not alter the general picture of polaron static properties (there is still an effective mass generation and self-trapping), they could possibly alter the dynamical response, which reveals such important information as how the impurities interact with their surroundings [7,8]. In this paper we would like to consider them in full detail and in different geometries with the special emphasis on strong coupling results thereby closing the gaps in the existing literature.

The paper is organized as follows. In the next section we formulate the problem and introduce all relevant quantities. Section 3 is devoted to the non-perturbative approach inspired by the classical random phase approximation (RPA). We explain the details of the implementation and discuss the special features pertinent to ultracold gas realizations. In Section 4 the calculation of the dynamical response function is accomplished using numerically exact quantum Monte Carlo (QMC) simulation technique. Section 5 contains a discussion of results and offers several avenues of further progress.

2. The model and observables

An effective low-energy Hamiltonian for a BEC-fermion mixture has the canonical Fröhlich form [3,6], which is written

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in terms of boson (described by the field operators $b_{\mathbf{k}}$) and fermion (denoted by $a_{\mathbf{q}}$) degrees of freedom:

$$H=\sum_q (E_q-\mu)\, a^\dagger_{\mathbf{q}} a_{\mathbf{q}}+\sum_k \omega_k b^\dagger_{\mathbf{k}} b_{\mathbf{k}}+\sum_q \sum_{k\neq 0} V_{\mathbf{k}}\, a^\dagger_{\mathbf{q}+\mathbf{k}} a_{\mathbf{q}}\,(b_{\mathbf{k}}+b^\dagger_{-\mathbf{k}}), \tag{1}$$

where the dispersion of fermions is $E_q=q^2/2m$, μ is their chemical potential

$$\omega_k=ck[1+(\xi k)^2/2]^{1/2} \tag{2}$$

is the dispersion of the phonon mode with an effective mass m_p, $c=(\sqrt{2}m_p\xi)^{-1}$ is the speed of sound in the condensate, and the coupling is given by $V_{\mathbf{k}}=\lambda[(\xi k)^2/((\xi k)^2+2)]^{1/4}$ with $\lambda=g_{\mathrm{IB}}\sqrt{N_0}$. g_{IB} is the effective interaction strength between the impurities and Bogoliubov excitations and can be adjusted by changing the particle density and/or the s-wave scattering length of collision processes of the impurity with the bosonic medium. ξ denotes the healing length of the condensate and is given by $\xi=1/\sqrt{8\pi a_{BB} n_0}$ where a_{BB} is the boson–boson s-wave scattering length and n_0 is the condensate density. We would like to point out that the model (1) is valid for not too strong boson–fermion scattering. As soon as the Bogolyubov approximation breaks down one has to work with the full interacting Hamiltonian [6]. Nonetheless, it was demonstrated previously that realistic boson–fermion mixtures turn out to show many details, which are adequately described by the strong coupling limit of the much simpler Fröhlich Hamiltonian [9]. That is why we concentrate on (1) throughout the paper. Yet another issue is that strong interactions might change the condensate fraction and thus influence the system parameters. As we focus on not too strong interactions we would like to neglect these effects.

One fundamental difference between the 'classical' semiconductor based electron–phonon coupled system and the one realized in ultracold mixtures is that the quantum gas system can be prepared in trapping potentials for fermions and bosons which might be of different shape and dimensionality. For that reason we shall later consider systems with different $E_{\mathbf{q}}$ and $\omega_{\mathbf{k}}$. Changing the shape of the trapping potential for the impurity in space and time, for instance by acceleration with respect to the BEC, which rests in the laboratory reference frame one induces the rearrangement of particles. In this way one can access the mobility of the impurity, which, like in the case of a Brownian motion, is the ultimate dynamical quantity of the particle [10]. In detail, the mobility is found from the velocity autocorrelation function, which translates into the current–current correlation function $\Pi(\mathbf{q},\omega)$ in the Matsubara representation:

$$\Pi_{\mu\nu}(\mathbf{q},\tau)=-\frac{1}{V}\langle T_\tau j^\dagger_\mu(\mathbf{q},\tau) j_\nu(\mathbf{q},0)\rangle, \tag{3}$$

with the current densities defined by

$$\mathbf{j}(\mathbf{q})=-\frac{1}{m\beta V}\sum_{\mathbf{k}}\left(\mathbf{k}+\frac{\mathbf{q}}{2}\right) a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}}. \tag{4}$$

This picture is very similar to the conventional polaron problem in semiconductors, where the principal quantity is the *momentum-dependent optical conductivity* [11]:

$$\mathrm{Re}[\sigma_{\mu\nu}(\mathbf{q},\omega)]=-\frac{e^2}{\omega}\mathrm{Im}[\Pi^R_{\mu\nu}(\mathbf{q},\omega)]. \tag{5}$$

Here by abuse of notation the subscript μ in the double sum indicates the spatial direction with respect to which the conductance is probed, that is $\mu\in\{x,y,z\}$. The superscript R denotes the retarded correlation function, which is obtained from the one in the Matsubara representation by the usual analytic continuation. From the perspective of a solid-state physicist, the optical conductivity computed at $\mathbf{q}=0$ describes the experimental conditions quite well, that is probing a sample with optical or X-ray photons does not lead to a substantial momentum transfer ($\Delta\mathbf{p}\approx 0$). In case of the RF-spectroscopy or the aforementioned experimental procedures in ultracold quantum gases, this is not necessarily the case. That is why throughout the paper we shall consider both $\mathbf{q}=0$ and finite $\mathbf{q}$ situations whenever possible.

Another experimental technique to access the impurity dynamics is the Bragg spectroscopy [12–17]. In a typical measurement the BEC is subject to two noncollinear laser beams with photons with wave vectors $\mathbf{k}_{1,2}$ and energies $\omega_{1,2}$. The fermionic atoms can then undergo a stimulated scattering absorbing the light from the beam 1 and emitting it into the laser field 2, thereby acquiring momentum and energy given by the differences of $\mathbf{k}_{1,2}$ and $\omega_{1,2}$. How much of the momentum and energy is absorbed by the BEC can then be mapped out by time-of-flight measurements after the trap release [17]. The absorption spectra are then directly related to the autocorrelation of the particle density (here $\mathbf{q}=\mathbf{k}_1-\mathbf{k}_2$):

$$\chi(\mathbf{q},\tau)=-\frac{1}{V}\langle T_\tau \rho^\dagger(\mathbf{q},\tau)\rho(\mathbf{q},0)\rangle, \tag{6}$$

where

$$\rho(\mathbf{q})=\sum_{\mathbf{k}} a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}}, \tag{7}$$

is the particle density operator. Very conveniently the *optical absorption spectrum*

$$R^{\mu\nu}_\Pi(\mathbf{q},\omega)=-\frac{1}{\pi}\mathrm{Im}\,\Pi^R_{\mu\nu}(\mathbf{q},\omega), \tag{8}$$

and *Bragg spectrum* (or Bragg spectral function), which we define as

$$R_\chi(\mathbf{q},\omega)=-\frac{1}{\pi}\mathrm{Im}\,\chi^R(\mathbf{q},\omega), \tag{9}$$

are related to each other in the following way (see Appendix A):

$$R_\chi(\mathbf{q},\omega)=\left(\frac{q}{\omega e}\right)^2 R^{\parallel}_\Pi(\mathbf{q},\omega), \tag{10}$$

where '$\parallel$' specifies the component in the direction of $\mathbf{q}$. Thus, once the current autocorrelation function is computed we have access to measurable quantities for both experimental schemes.

Now we would like to translate the autocorrelation function (3) into the operator language of the original Hamiltonian. From now on we skip the vector notation since we would like to restrict ourselves to 1D only. Experimentally, this can be motivated by the use of quantum gases in reduced dimensions. Although in translationally invariant 1D systems no BE condensation is possible, in a realistic experimental situation there is always a confinement potential which facilitates a condensation. That is why it is legitimate to work in that picture. In energy-momentum representation we then obtain

$$\Pi(q,i\omega_n)=-\frac{1}{V}\int_0^\beta d\tau e^{i\omega_n\tau}\langle T_\tau j^\dagger(q,\tau) j(q,0)\rangle \tag{11}$$

$$\begin{aligned}
&=\frac{e^2q^2}{(i\omega_n)^2m^3V}\sum_k\left(3k^2+\frac{q^2}{4}\right)\langle a^\dagger_k a_k\rangle-\frac{e^2}{(i\omega_n)^2m^2V}\sum_{q'}q'(q+q')V^*_{q'}\langle B_{q'}\rho^\dagger(q')\rangle\\
&-\frac{e^2}{(i\omega_n)^2m^2V}\int_0^\beta d\tau e^{i\omega_n\tau}\left[\frac{q^2}{m^2}\sum_{kk'}\left(k+\frac{q}{2}\right)^2\left(k'+\frac{q}{2}\right)^2\langle T_\tau a^\dagger_k(\tau)a_{k+q}(\tau)a^\dagger_{k'+q}a_{k'}\rangle\right.\\
&-\frac{q}{m}\sum_{kq'}V_{q'}q'\left(k+\frac{q}{2}\right)^2\langle T_\tau B^\dagger_{q'}a^\dagger_k(\tau)a_{k+q}(\tau)\rho(q+q')\rangle\\
&-\frac{q}{m}\sum_{kq'}V^*_{q'}q'\left(k+\frac{q}{2}\right)^2\langle T_\tau B_{q'}\rho^\dagger(q+q',\tau)a^\dagger_{k+q}a_k\rangle\\
&\left.+\sum_{q'q''}V^*_{q'}V_{q''}q'q''\langle T_\tau B_{q'}(\tau)B^\dagger_{q''}\rho^\dagger(q+q',\tau)\rho(q+q'')\rangle\right],
\end{aligned} \tag{12}$$

where $B_q=b_q+b^\dagger_{-q}$. This formula will be evaluated in the analytical computations of the following section, while for the QMC simulation of Section 4 the current and density correlations would be computed in a slightly different way.