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Field emission from finite barrier quantum structures

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ABSTRACT

We study field emission from various finite barrier quasi-low dimensional structures, taking image force into account. To proceed, we first formulate an expression for field emission current density from a quantum dot. Transverse dimensions of the dot are then increased in turn, to obtain current densities respectively from quantum wire and quantum well with infinite potential energy barriers. To find out field emission from finite barrier structures, the above analysis is followed with a correction in the energy eigen values. In course, variations of field emission current density with strength of the applied electric field and structure dimensions are computed considering *n*-GaAs and *n*-GaAs/$Al_xGa_{1-x}As$ as the semiconductor materials. In each case, the current density is found to increase exponentially with the applied field, while it oscillates with structure dimensions. The magnitude of the emission current is less when the image force is not considered, but retains the similar field dependence. In all cases, the field emission from infinite barrier structures exceeds those from respective finite barrier ones.

1. Introduction

Application of a high electric field ($\sim 10^9$ V/m) on different metals and quantum structures results in Fowler Nordheim tunneling [1–7]. Image force effects on such tunneling phenomena [8,9] have been well investigated. Most of these works deal with infinite potential barriers. In this communication, we formulate field emission current density from infinite quantum dot (QD) of rectangular cross-section in the presence of image charge originating due to discontinuity in permittivity at the well boundary [10]. The image charge lowers the potential energy barrier to the electrons and consequently modifies the transmission coefficient. WKB approximation is used here to estimate the probability of an electron to tunnel through the modified barrier. The transverse dimensions of the dot are also extended in order to study the field emission current density from other quasi low dimensional (QLD) structures like quantum wire and quantum well (QW). The potential energy barrier in a practically realizable quantum structure is indeed far from infinity (even less than 1 eV). Wavefunctions appropriate for such finite barrier QLD structures are, therefore, introduced in the above analysis of field emission. To make the influence of image force on the resulting emissions clear, above studies are repeated in absence of image force.

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2. Theory

A high electric field F is applied to a QD of rectangular cross-section along $-z$ direction. The dot is assumed to have width d_1 along z direction, thickness d_2 along y direction and length d_3 along x direction.

In this section we intend to find an analytical expression for field emission from such a QD including image force. For this we use the general expression of field emission current density [3]

$$J = \frac{1}{2}\sum_n \sum_m \sum_l e v_n \Delta n_{mnl} t, \tag{1}$$

where, Δn_{mnl} is the volume density of electrons given by

$$\Delta n_{nml} = \frac{2}{d_1 d_2 d_3 \left(1 + \exp\left(\frac{E_{nml} - E_F}{k_B T}\right)\right)}, \tag{1a}$$

and v_n, their velocity in the direction of the applied field is

$$v_n = \frac{\hbar}{m^*}\left(\frac{n\pi}{d_1}\right) \tag{1b}$$

with E_F being the Fermi energy and E_{nml} being the electron energy level given as

$$E_{nml} = \frac{\hbar^2}{2m^*}\left[\left(\frac{n\pi}{d_1}\right)^2 + \left(\frac{m\pi}{d_2}\right)^2 + \left(\frac{l\pi}{d_3}\right)^2\right]$$

To estimate transmission coefficient t, dot material is assumed to be heavily doped so that band bending can be ignored there and the simplified band diagram presented in Fig. 1 can be used. In the barrier region, potential energy of an electron at a distance z from

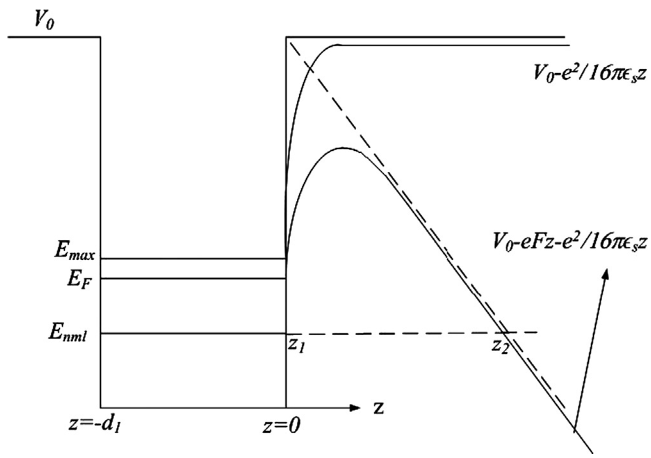

Fig. 1. Schematic band diagram of the QD along z direction.

the dot surface is then given by

$$V(z)=\left(V_0-eFz-\frac{e^2}{16\pi\int_s z}\right) \tag{2}$$

where V_0 and ϵ_s are respectively height of the potential energy barrier and permittivity of the semiconductor barrier material.

The transmission coefficient $t=\exp(-(2/\hbar)\int_{z_1}^{z_2}|p_z|dz)$, obtained by using WKB approximation, will take the form

$$t=\exp\left[-\frac{2}{\hbar}\int_{z_1}^{z_2}\sqrt{2m^*\left(V_0-E_{nml}-eFz-\frac{e^2}{16\pi\int_s z}\right)}\right] \tag{3}$$

with the limits of integration $z_{1,}$ z_2 to be found from Fig. 1.

The above equation is rewritten as

$$t=\exp\left[-2\left(\frac{2m^*}{\hbar^2}\right)^{1/2}\frac{W_{nml}}{2\sqrt{eF}}\int_{z_1}^{z_2}\right.$$
$$\left.\times\left\{\left(\frac{1}{z}\right)\left(1-\frac{e^3F}{4\pi\int_s W_{nml}^2}-\left(\frac{2eFz}{W_{nml}}-1\right)^2\right)\right\}^{1/2}dz\right], \tag{4}$$

where $W_{nml}=V_0-E_{nml}$.

Defining $y=(e\sqrt{eF}/\sqrt{4\pi\int_s W_{nml}})$ and subsequently $b=(2eFz/W_{nml})-1$, we can write (4) as

$$t=\exp\left[-2\left(\frac{2m^*}{\hbar^2}\right)^{1/2}\frac{W_{nml}}{2\sqrt{eF}}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\sqrt{\frac{W_{nml}}{2eF}}\left(\frac{1-y^2-b^2}{1+b}\right)^{1/2}db\right], \tag{5}$$

which can be further simplified to yield

$$t=\exp\left[-\left(\frac{m^*}{\hbar^2}\right)^{1/2}\frac{{W_{nml}}^{3/2}}{eF}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\left(\frac{1-y^2-b^2}{1+b}\right)^{1/2}db\right]. \tag{6}$$

We next substitute $1-y^2=a^2$ and use the conversion

$$\left(\frac{a^2-b^2}{1+b}\right)^{1/2}=\frac{\frac{1}{3}\left(a^2-2b-3b^2\right)+\frac{2}{3}(a^2-1)+\frac{2}{3}(b+1)}{\left(-b^3-b^2+a^2b+a^2\right)^{1/2}}$$

to obtain the integral of (6) as

$$I=\frac{2}{3}\int_{-a}^{a}\frac{a^2-b^2}{\left(-b^3-b^2+a^2b+a^2\right)^{1/2}}db.$$

By means of the substitution $b=a-2a\sin^2\alpha$, I can be reduced to

$$I=\frac{4}{3}\left[\frac{a^2-1}{\sqrt{1+a}}K(\lambda)+\sqrt{1+a}E(\lambda)\right],$$

where $\lambda=\sqrt{2a/1+a}$, $K(\lambda)$ and $E(\lambda)$ are complete elliptical normal integrals of first and second kind respectively.

Therefore, considering

$$\theta(a)=\frac{1}{\sqrt{2}}\left(\frac{1-a^2}{\sqrt{1+a}}K(\lambda)+\sqrt{1+a}E(\lambda)\right),$$

the field emission current density from a QD can finally be expressed, in presence of image force as

$$J=\frac{e\pi\hbar}{m^*d_1^2d_2d_3}\sum_n n\sum_m\sum_l\frac{\exp\left[-\frac{4}{3}\left(\frac{2m^*}{\hbar^2}\right)^{1/2}\frac{(V_0-E_{nml})^{3/2}}{eF}\theta(a)\right]}{1+\exp\left(\frac{E_{nml}-E_F}{k_BT}\right)}. \tag{7}$$

If thickness d_2 of the QD is increased such that the quantization effect does not prevail along that direction, the dot can be treated as a quantum well wire (QWW). The summation over l in (7) is then replaced by integration within limits of 1 and $l_{\max}$, where $l_{\max}$ corresponds to the quantum number of the highest occupied energy level. The resulting expression becomes

$$J=\frac{e\pi\hbar}{m^*d_1^2d_2d_3}\sum_n n\sum_m\int_1^{l_{\max}}\frac{\exp\left[-\frac{4}{3}\left(\frac{2m^*}{\hbar^2}\right)^{1/2}\frac{(V_0-E_{nml})^{3/2}}{eF}\theta(a)\right]}{1+\exp\left(\frac{E_{nml}-E_F}{k_BT}\right)}dl \tag{8}$$

It is clear from Fig. 1 that the tunneling probability is maximum at the highest occupied level $E=E_{max}$, and falls rapidly for lower energy levels due to increase in barrier thickness. Therefore, the exponential term in relation to the transmission coefficient in (8) is simplified using Taylor's series expansion and terms only up to first derivative are retained. To proceed further, the following substitutions are made:

$$y_0=\frac{e\sqrt{eF}}{\sqrt{4\pi\int_S(V_0-E_{\max})}}$$

$$a_0=\sqrt{1-y_0^2}$$

$$t_0=\exp\left[-\frac{4}{3}\left(\frac{2m^*}{\hbar^2}\right)^{1/2}\frac{(V_0-E_{\max})^{3/2}}{eF}\theta(a_0)\right]$$

$$\eta=-2\left(\frac{2m^*}{\hbar^2}\right)^{1/2}\frac{(V_0-E_{\max})^{1/2}}{eF}\left[\theta(a_0)-\frac{2}{3}\left(a_0-\frac{1}{a_0}\right)\frac{d\theta}{da}\bigg|_{a_0}\right].$$

The exponential term in consideration is now reduced to $t_0\exp[-\eta(E_{\max}-E_m-E_n-E_l)]$ and (8) takes the form

$$J=\frac{et_0}{\sqrt{2m^*}d_1^2d_2}\sum_n n\sum_m\int_0^\infty\frac{1}{\sqrt{E_l}}\frac{\exp[-\eta(E_{\max}-E_m-E_n-E_l)]}{[1+\exp(E_{nml}-E_F/k_BT)]}dE_l, \tag{9}$$

which can further be expressed as

$$J=\frac{et_0}{\sqrt{2m^*}d_1^2d_2}\sum_n n\sum_m\exp[-\eta(E_{\max}-E_m-E_n)]$$
$$\times\int_0^\infty\frac{1}{\sqrt{E_l}}\frac{\exp(\eta E_l)}{1+\exp(E_{nml}-E_F/k_BT)}dE_l. \tag{10}$$

For a very low temperature, all the energy levels up to the Fermi level can be assumed to be fully occupied, and the above equation is then modified to

$$J=\frac{et_0}{\sqrt{2m^*}d_1^2d_2}\sum_n n\sum_m\exp[-\eta(E_F-E_m-E_n)]\int_0^{E_F-E_{mn}}\frac{1}{\sqrt{E_l}}\exp(\eta E_l)dE_l. \tag{11}$$

Use of the power series expansion for the term $\exp(\eta E_l)$, followed by some mathematical manipulation, yields the integral