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ORIGINAL ARTICLE

Quadratic and nonlinear programming problems solving and analysis in fully fuzzy environment

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Abstract This paper presents a ***comprehensive*** methodology for solving and analyzing quadratic and nonlinear programming problems in fully fuzzy environment. The solution approach is based on the Arithmetic Fuzzy Logic-based Representations, previously founded on normalized fuzzy matrices. The suggested approach is generalized for the fully fuzzy case of the general forms of quadratic and nonlinear modeling and optimization problems of both the *unconstrained* and *constrained* fuzzy optimization problems. The constrained problems are extended by incorporating the suggested fuzzy logic-based representations assuming complete fuzziness of all the optimization formulation parameters. The robustness of the optimal fuzzy solutions is then analyzed using the recently newly developed ***system consolidity index***. Four examples of quadratic and nonlinear programming optimization problems are investigated to illustrate the efficacy of the developed formulations. Moreover, consolidity patterns for the illustrative examples are sketched to show the ability of the optimal solution to withstand any system and input parameters changes effects. It is demonstrated that the geometric analysis of the consolidity charts of each region can be carried out based on specifying the type of consolidity region shape (such as elliptical or circular), slope or angle in degrees of the centerline of the geometric, the location of the centroid of the geometric shape, area of the geometric shape, lengths of principals diagonals of the shape, and the diversity ratio of consolidity points. The overall results demonstrate the consistency and effectiveness of the developed approach for incorporation and implementation for fuzzy quadratic and nonlinear optimization problems. Finally, it is concluded that the presented concept could provide a comprehensive methodology for various quadratic and nonlinear systems' modeling and optimization in fully fuzzy environments.

1. Introduction

The fuzzy systems in general can be designed to supplement the interpretation of uncertainties for real world random phenomenon. The fuzzy decision techniques allow collecting subjective data on what analyst perceive as relevant risk factors, and their relative importance, and to relatively build *individual*

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or *group* models for risk assessment. In dealing with these types of problems, several relevant techniques can be applied such as fuzzy mathematical programming, stochastic programming, fuzzy neural networks, fuzzy genetic algorithms, fuzzy particle swarm techniques, and fuzzy ant colony approach. Usually the available techniques can handle either *single objective* or *multi-objectives* formulations [1–4].

Fuzzy logic optimization is an extension of global optimization techniques operating in fuzzy environment. The classifications of the common fuzzy logic optimization techniques are elucidated in Fig. 1, [5–8]. In general, these methods represent an extension of global optimization techniques in fuzzy environment. Examples of the common fuzzy logic optimization approaches reported in the literature are fuzzy mathematical programming, fuzzy evolutionary algorithms, and fuzzy operations research techniques.

Most of these fuzzy optimization problems formulations are based on the characteristics of fuzzy goal, fuzzy constraints and fuzzy coefficients. In fuzzy environment, mathematical optimization models have to take into consideration of both flexible constraints and vague objective function. Many fuzzy optimization problems are formulated based on this conjunction.

In real life problems, various variables could have different fuzzy membership functions, fuzzy intervals and fuzzy matrices [9–12]. A normalization step has to be applied in order to unify these membership functions in one combined (compromising) function for each problem that can be applied for all situations as presented by Gabr and Dorrah [13–19]. There are many other areas in which fuzzy modeling and optimization can be used including the following: traffic systems, robotics, computers, industrial processes, biology and medicine, projects management and business. This list is by no means exhaustive. Virtually any computer decision-making system has the potential to benefit from the application of fuzzy logic for decision making under uncertainty.

2. The proposed methodology

2.1. The consolidity index

In this paper, a comprehensive methodology is presented for solving and analyzing general classes of *non-constrained* and *constrained* quadratic and nonlinear programming optimization problems in open fully fuzzy environment.[1] The approach used is by applying the arithmetic and visual fuzzy-based representation developed on the basis of normalized fuzzy matrices [13–19]. The robustness of the optimal fuzzy solutions will be then tested by the ***system consolidity index*** as defined in Appendix A [19–25].

Consolidity (the act and quality of consolidation) is a measured by the systems output reactions versus combined input/system parameters reaction when subjected to varying environments and events [1–3]. Moreover, consolidity can govern the ability of systems to withstand changes when subjected to incurring events or varying environments. In fact, consolidity is the scaling factor of managing system changes.

2.2. The consolidity chart

The analysis of the ***consolidity chart*** (or patterns chart) will be based on constructing the best geometric region that appropriately embodies all the various consolidity points obtained through the overall output fuzziness magnitude $|F_O|$ at the y-axis versus the overall combined input and system fuzziness magnitude $|F_{I+S}|$ at the x-axis. The definition of both $|F_O|$ and $|F_{I+S}|$ are given in Appendix A [22]. Such geometric region could follow many shapes such as the elliptical, circular or other forms. Furthermore, it can be analyzed for its geometric features as presented in the following table:

Symbol	Description
R	Type of consolidity region shape (elliptical, circular, or others)
Region class	Types of region classes are as follows: (i) consolidated, (ii) neutrally consolidated, (iii) unconsolidated, (iv) quasi-consolidated, (v) quasi-unconsolidated, or (vi) mixed-consolidated
S	Slope or angle in degrees of the S (degrees) = $\tan^{-1}$ (overall consolidity index)
$C=(x,y)$	Coordinates of the centroid of the geometric shape R
A	Area of the geometric shape of R in pu^2
l_1	Length of major diagonal of region (pu)
l_2	Length of minor diagonal of region (pu)
l_2/l_1	Diversity ratio of consolidity points (unitless)

Two case studies of the consolidity chart regions of elliptical and circular types are shown in Fig. 2. The analysis of the two cases can be summarized as follows:

Symbol	Meaning	Case I	Case II
R	Shape type	Elliptical	Circular
Region class	Region location	Unconsolidated	Consolidated
S	Slope	63.05°, or $\tan^{-1}$ (1.9667)	21.80°, or $\tan^{-1}$ (0.4000)
$C=(x,y)$	Centroid	(3.0,6.0)	(5.0,2.0)
A	Area (pu^2)	11.5	6.6
l_1	Length of major diagonal (*pu*)	5.75	2.90
l_2	Length of minor diagonal (*pu*)	2.55	2.90
l_2/l_1	Diversity ratio	0.4435	1.0000

In the above analysis, the position of the centroid $C=(x,y)$ (upward or downward) within main centerline depends mainly on the nature of the affected input fuzzy influences which are particular for each specific application. Higher values of such centroids mean higher fuzzy input effects or influences. In addition, a better system from the consolidity chart point of view is the one with smaller slope, smaller area A and smaller diversity ratio l_2/l_1.

[1] An "***open Fully Fuzzy Environment***" is defined as that all fuzzy levels can freely change all over the*positive* and *negative* values of the environment. A subclass of this environment is *bounded fuzzy environment* where all fuzzy levels can only change within restricted positive and negative ranges of the environment.