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Effects of buoyancy and thermal radiation on MHD flow over a stretching porous sheet using homotopy analysis method

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Abstract This paper investigates the theoretical influence of buoyancy and thermal radiation on MHD flow over a stretching porous sheet. The model which constituted highly nonlinear governing equations is transformed using similarity solution and then solved using homotopy analysis method (HAM). The analysis is carried out up to the 5th order of approximation and the influences of different physical parameters such as Prandtl number, Grashof number, suction/injection parameter, thermal radiation parameter and heat generation/absorption coefficient and also Hartman number on dimensionless velocity, temperature and the rate of heat transfer are investigated and discussed quantitatively with the aid of graphs. Numerical results obtained are compared with the previous results published in the literature and are found to be in good agreement. It was found that when the buoyancy parameter and the fluid velocity increase, the thermal boundary layer decreases. In case of the thermal radiation, increasing the thermal radiation parameter produces significant increases in the thermal conditions of the fluid temperature which cause more fluid in the boundary layer due to buoyancy effect, causing the velocity in the fluid to increase. The hydrodynamic boundary layer and thermal boundary layer thickness increase as a result of increase in radiation.

1. Introduction

Stretching sheet is essential in industrial processes. This process is usually following with heat and mass transfer aspects. The influence of radiation on MHD flow and heat transfer in industrial and technological areas occurs at high temperatures and knowledge of radiation heat transfer becomes very vital for design of pertinent equipment. For production of plastic sheets, gas turbines, missiles, space vehicles aircraft, nuclear power plants, satellites and foils see [1–5]. The influence of variable thermal conductivity and radiation on the flow and heat transfer was carried out by Mahmoud [6]. Pal and Mondal [7] analyse the effect of variable viscosity on MHD non-Darcy boundary layer flow and heat transfer features in an incompressible electrically conducting fluid. Magnetohydrodynamic (MHD) based Nanofluids with Natural convection through porous sheet were presented by

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Nomenclature

a	constant
$B(x)$	magnetic field
B_0	applied magnetic induction
f	dimensionless stream function
g	acceleration due to gravity
Gr	Grashof number
k	fluid thermal conductivity
m	velocity exponent parameter
M	magnetic parameter
n	temperature exponent parameter
c_p	specific heat at constant pressure
Pr	Prandtl number
q_r	thermal radiation
Re_x	local Reynolds number
T	fluid temperature
$T_w(x)$	temperature of the stretching sheet
T_∞	free stream temperature
u, v	fluid velocity components along the x and y directions, respectively
$U(x)$	velocity of the stretching sheet
x, y	vertical and horizontal directions

Greek Symbols

α	thermal diffusivity
β	volumetric expansion coefficient
η	similarity variable
λ	buoyancy or mixed convection parameter
θ	dimensionless temperature
μ	dynamic viscosity
v	kinematic viscosity
ρ	fluid density
σ	electrical conductivity
τ_w	skin friction
ψ	stream function

Zeeshan et al. [8]. Also, El-Aziz [9] studied influence of thermal radiation on the flow over an unsteady stretching surface and Pal and Mondal [10] extended the work to ohmic dissipation and thermal effects. Pal and Chatterjee [11] work on micropolar fluid over a stretching sheet in a non-Darcian porous medium on MHD boundary layer flow. Sheikholeslami et al. [12] consider Lorentz force to investigate CuO-water nanofluid flow and convective heat transfer. An extension was made by Sheikholeslami et al. [13] to thermal radiation on nanofluid flow using two phase model. Rashidi et al. [14] consider stream wise transverse magnetic fluid flow in porous medium with heat transfer. Important analysis concerning the Buoyancy and MHD boundary layer flow over stretching sheet was carried out by Makinde et al. [15] and Nadeen et al. [16]. To study the stability of the problem understudy see Hajmohammadi and Nourazar [17] by using a semi-analytical method and the new "gradient energy method" has been used by Hajmohammadi and Nourazar [17] and Hajmohammedi et al. [18]. In view of this some other researchers use semi-analytical approach on the problems of variable thermal conductivity such as the work of Khan et al. [19], Gul et al. [20] and Hajmohammadi and Nourazar [21].

The effect of radiation on MHD steady asymmetric flow was considered by Makinde [22] of an electrically conducting fluid over a stretching porous sheet. Khan et al. [23] study Oldroyd fluid with inclined magnetic field in the presence of heat transfer. The flow and heat transfer phenomenon in a power law fluid over a porous stretching sheet with effect of magnetic field was considered by researchers such as in [24–27]. MHD flow of an incompressible viscous and electrically conducting fluid over a vertical stretching sheet embedded in a porous medium was considered by Pal and Mondal [3], Mohammed [4], Shateyi et al. [28]. The flow of variable electric conductivity inclined impermeable flat plate subject to a uniform surface heat flux boundary condition was carried out by Rahman et al. [29]. Looking at the effects of viscous dissipation are the works of Hajmohammadi and Nourazar [30] and Hajmohammadi et al. [31]. For effect of the buoyancy force and thermal radiation in MHD boundary layer visco-elastic fluid over continuously moving stretching surface in a porous surface see ([32–36]). Seddeck [37] analyses thermal-diffusion and the diffusion-thermo effects on the mixed free-forced convective and mass transfer steady laminar boundary layer flow over an accelerating surface with a heat source in the presence of suction and blowing. The influence of MHD and temperature dependent of non-Newtonian nanofluid was studied by Ellahi [38]. Mahmoud [6] studied flow and heat transfer of an incompressible viscous electrically conducting fluid over a continuously moving vertical infinite plate with suction and heat flux in the presence of radiation. An extension to thermal radiation combined buoyancy and suction/blowing was carried out by Sheteyi [39].

In the present paper, the object of this study was to present a unified approach to solving the MHD flow due to influence of buoyancy and thermal radiation over a stretching porous sheet using homotopy analysis method (HAM). There have been several approaches on theoretical models developed to describe buoyancy and thermal radiation on hydrodynamic magnetic field flow over a stretching sheet. However, to the best of our knowledge, no investigation has been made yet to analyse the effects of buoyancy and thermal radiation on MHD flow over a stretching porous sheet using HAM. The non-linear differential equations are solved using HAM proposed by Liao [40] and Liao and Tan [41].

2. Mathematical analysis

In our present study on effect of thermal radiation, we assumed the flow to be laminar and stable. Consider a steady two-dimensional laminar flow of a viscous, incompressible and electrically conducting fluid past a stretching sheet. The stretching sheet is assumed to be permeable in order to give way for possible wall fluid suction/injection. By using two equal and opposite forces along the horizontal direction, with the influence of uniform magnetic field normal to the plate, the uniform magnetic field as result of velocity of the electrically