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Similarity solutions of quasi three dimensional power law fluids using the method of satisfaction of asymptotic boundary conditions

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Abstract The work presented in this paper is focused on deductive group-theoretic transformations to develop the similarity solution of steady, laminar, incompressible quasi three dimensional boundary layer flow governing power law fluid. The application of one-parameter group reduces the number of independent variables to one and consequently the system of governing, highly non-linear partial differential equations reduces to a self similar, non-linear ordinary differential equation with appropriate auxiliary conditions. The numerical solution for a power law fluid considered for small cross flow is obtained systematically using MSABC in dimensionless form.

1. Introduction

The boundary layer flow occurs over various aerodynamic configurations such as wings, missiles, fuselage forms or channel flows. For the flow over wings and channels, the boundary layer generally develops along a line on the surface (e.g. leading edge). The development of boundary layer is useful for efficient design procedures of both internal turbo machine components and external surface components. The gross allowances for the boundary layer were possible to make, using empirical relations along with results predicted from two dimensional boundary layer theory. But this method does not seem much promising for three dimensional boundary layer characteristics. As a result, a great deal of experimental and theoretic research on three dimensional flows has been carried on in recent years. But still a need exists for theoretic analyses which will lead to the predictions of boundary layer behavior.

Theoretical research has been restricted because of the complex nature of the equations describing the flow, while in a practical problems, the boundary layer is turbulent. Some success, however, has been achieved in the analysis of the three dimensional laminar equations. There is always a need to search for exact solutions of the laminar, incompressible, boundary layer equations for special types of main stream flows. From a physical point of view, these boundary layer equations have the capacity to admit a large number of invariant solutions which are known as similarity solutions. These invariant solutions are meant to reduce nonlinear partial differential equations of the boundary layer to a system of ordinary differential equations.

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Nomenclature

u, v, w	velocity component in the boundary layer along x, y, z-axis respectively	Re	Reynolds number
τ_{yx}, τ_{yz}	the two non-vanishing components of the shear tensor	ψ	arbitrary mathematical function
x, y	Cartesian coordinates	G, $\overline{G}$, g	arbitrary constants
m, n	parameters in the mathematical model of a power-law fluid	Π, €, £	arbitrary constants
U, W	velocity component in the main flow along x, z-axis respectively	$N_1 \ldots N_4$	arbitrary constants
ρ	density of the fluid	a, $\propto_{11} \ldots \propto_{81}$	arbitrary constants
L, U_0	characteristic length and velocity respectively	β, $\beta_{11} \ldots \beta_{81}$	arbitrary constants
		η	independent variable in transformed ordinary differential equation
		$F_1 \ldots F_4$	dependent variables in transformed ordinary differential equation

For any non-Newtonian fluid, two entities are important viz. the mathematical structure of the shearing stress and the rate of shear. Such a mathematical formulation is indeed a difficult task. Since great diversity is found in the physical structure of non-Newtonian fluids, it is difficult to recommend a single constitutive equation to describe them. When shear stress is an arbitrary function of the velocity gradient, the non-Newtonian fluid model represents Visco-Inelastic behavior which is observed in several fluids including Newtonian fluids, Prandtl fluids, Prandtl–Eyring fluids, Power-law fluids, Eyring fluids, Sisko fluids, Sutterby fluids, Ellis fluids, Williamson fluids, Reiner–Philippoff fluids, Powell Eyring fluids. To investigate the non-Newtonian effects, similarity solutions play an important role because being exact solutions, they serve as a reference to check approximate solutions.

Hansen and Herzig [1–3] has developed three dimensional boundary layer equations for the flow past flat surface for Cartesian, Curvilinear and Polar coordinate system and derived similarity solution for all three cases. But all these past cases were limited to Newtonian fluids only. Schowalter [4] was probably the first to introduced similarity solution for three dimensional non-Newtonian Power law fluids using separation of variable method. Na and Hansen [5] have obtained similarity solution for three dimensional boundary layer equations for non-Newtonian Power law fluids by linear and spiral group of transformations. They have also obtained similarity solution for small cross flow geometry. Timol and Kalthia [6] have carried out similarity analysis of three dimensional boundary layer equations of non-Newtonian fluids and integrated similarity equations for Reiner Philippoff fluids. Pakdemirli [7,8] and his co-workers have worked out similarity solutions for three dimensional boundary layer flow of non-Newtonian Power law fluids using scaling and spiral group of transformations. However, in all these cases, similarity equations were highly nonlinear coupled differential equations and remain numerically unsolved.

Nowadays, many techniques are available for similarity analysis. Among them, the similarity methods which invoke the invariance under the group of transformations are known as group theoretic methods. These methods are more recent and are mathematically elegant; hence they are widely used in different fields. The group theoretic methods involve mainly two different types of groups of transformations, namely, assumed group of transformations and deductive group of transformations. The linear group transformations, scaling group transformations, spiral group transformations are the assumed group of transformations and are mainly due to Birkhoff [9] and Morgan [10] where as the deductive group of transformations can be further classified into two groups: finite group of transformations Moran and Gaggioli [11] and infinitesimal group of transformation Bluman and Cole [12], Bluman and Kumai [13].

The main drawback of similarity methods based on the assumed group of transformation at the outset of the analysis is that, the resulting similarity solutions are restrictive and may sometimes lead to wrong conclusion that the similarity transformations does not exist. On the other hand, the similarity methods based on general group of transformation are more systematic and lead to a number of similarity solutions. Out of these, the deductive group theoretic method provides a powerful tool because it is not based on linear operators, superposition, or any other aspect of linear solution techniques. Therefore, this method can successfully be applied to nonlinear differential models.

Recently, deductive group of transformation has been successfully applied to various non-linear two dimensional flow problems by Abd-el-Malek et al. [14], Parmar and Timol [15], Adnan et al. [16] and Darji and Timol [17]. The objective of present investigation was to apply the deductive group method based on general group of transformation to derive similarity solutions for steady, quasi three dimensional incompressible laminar boundary layer flows of non-Newtonian Power law fluid. We treat Ostwald-de model of Power law fluids as it is most widely used model to exhibit non-Newtonian behavior in fluids and to predict shear thinning and shear thickening behavior.

Solution of the final similarity equations in general, requires the application of numerical techniques. Little work has been done on solving these equations. Thus we have made an attempt to find the numerical solution of nonlinear coupled ordinary differential equations.

2. Governing equations

The governing differential equations for the boundary layer flow of the generalized non-Newtonian fluid are given as [5]