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ORIGINAL ARTICLE

Effect of heat sink/source on peristaltic flow of Jeffrey fluid through a symmetric channel

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Abstract The peristaltic flow and heat transfer through a symmetric channel in the presence of heat sink/source parameter have been analyzed in this paper. It also deals with the effect of the natural convection coefficient in the momentum equation. Low Reynolds number and small wave number approximation are used to convert the non-linear partial differential equations into the non-linear ordinary differential equations. In order to solve the governing model, perturbation method has been chosen by taking α (material parameter) as a small parameter. Expressions have been obtained for temperature, velocity, stream function, pressure rise and frictional forces. The features of the flow characteristics are analyzed by plotting graphs and the results are discussed in details. It has been observed that velocity increases with an increase of α (material parameter). The peristaltic pumping and in the copumping region the pumping rate decreases by increasing the value of α (material parameter). The size of the trapped bolus decreases by increasing the value of α (material parameter). The temperature profile increases by increasing the value of β_1 (heat sink/source parameter).

1. Introduction

Heat transfer occurs in almost all branches of Engineering and Biosciences. Also, it has many industrial, chemical, residential and commercial applications. It is well-known heat transfer follows the first law of thermodynamics.

Peristaltic transport is a means of fluid flow in elastic path by the way of contraction and expansion processes. This flow induces progressive waves along the elastic path. Peristaltic transport is an inbuilt mechanism in many biological systems including human body such as: transportation of biofluids, spermatozoa in the ducts efferent of the male reproductive tract, ovum in the female fallopian tube, lymph in the lymphatic vessel and also in many other glandular ducts. Technical roller and finger pumps also operate according to the peristaltic principle. Further it is also used in transporting sensitive or corrosive fluids, sanitary fluids and noxious fluids in nuclear industry. The first systematic investigation of

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peristaltic measure has been carried out by Latham [1]. Later on more researches carried out this work [2–5].

For the occurrence of heat transfer with peristaltic flow, there must be transport of fluid by contraction and expansion involving kinetic energy. The peristaltic propulsion will result into the transfer of heat. Some other examples include: drying, evaporation, cooling, combustion of fuel droplets and ablation cooling, vehicles sourcing, reentry and even ordinary vents such as rain, snow melts and hail. Keeping in view of its importance and real world validity several researchers [6–10] have studied the effects of heat transfer on peristaltic transport of Newtonian and non-Newtonian fluids.

Some recent researches are as follows: Vasudev et al. has investigated the effects of heat transfer on the peristaltic flow of Jeffrey fluid through a porous medium in a vertical annulus [11]. Khan et al. have presented the peristaltic transport of a Jeffrey fluid with variable viscosity through a porous medium in an asymmetric channel [12]. Akram et al. have discussed the significance of Nanofluid and partial slip on the peristaltic transport of a Non-Newtonian fluid with different waveforms [13]. Pandey has presented the unsteady model of transportation of Jeffrey fluid by peristalsis [14]. Akbar et al. have analyzed the thermal and velocity slip effects on the MHD peristaltic flow with carbon nanotubes in an asymmetric channel: application of radiation therapy [15]. Noreen has studied the numerical solution of the effects of induced magnetic field on Jeffrey six-constant fluid with peristaltic flow [16].

Also several works available in the literature deal with passage arrangement optimization in plate-fin heat exchangers such as [25–27].

This paper considers Jeffrey six-constant fluid with heat transfer. The Jeffrey model is relatively simple linear model using time derivatives instead of convected derivatives e.g., the Oldroyd-B model, and it represents a rheology different from Newtonian fluid. The whole analysis has been carried out in a moving frame of reference. The governing equations of fluid flow are solved subject to relevant boundary conditions. Perturbation solutions of pressure gradient, stream function and temperature distribution are obtained. Graphs are plotted for different parameters.

2. Indispensable equations

The constitutive equations for Jeffrey six-constant fluid model are [17]

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\rho \frac{d\mathbf{V}}{dt} = \nabla \cdot \mathbf{T}' + \rho \mathbf{b_f} + \mathbf{R}, \tag{2}$$

$$\rho C_p \frac{dT}{dt} = k\nabla^2 T + Q_0, \tag{3}$$

and the Cauchy stress Tensor is

$$\mathbf{T}' = -p\mathbf{I} + \mathbf{S}, \tag{4}$$

$$\mathbf{S} + \lambda_1\left[\frac{d\mathbf{S}}{dt} - W\mathbf{S} + \mathbf{S}W + \widetilde{a}(\mathbf{S}D + D\mathbf{S}) + \widetilde{b}\mathbf{S} : DI + \widetilde{c}Dtr\mathbf{S}\right] = 2\mu\left[D + \lambda_2\left(\frac{dD}{dt} - WD + DW + 2\widetilde{a}DD + \widetilde{b}D : DI\right)\right], \tag{5}$$

$$\mathbf{L} = \text{grad } \mathbf{V}, \tag{6}$$

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T. \tag{7}$$

Here, $\mathbf{V}$ is the velocity vector, $\mathbf{b_f}$ is the body force (assumed to be zero), $\mathbf{R}$ is the Darcy's resistance (assumed to be zero), ρ is the fluid density, $\mathbf{S}$ is the extra stress tensor, $\mathbf{T}'$ is the Cauchy stress tensor, C_p is the specific heat, T is the temperature, k is the constant thermal conductivity, $Q_0 > 0$ is the heat source and $Q_0 < 0$ is the heat sink, p is the pressure, $D = \frac{(\nabla \mathbf{V}^t + \nabla \mathbf{V})}{2}$ is the symmetric part of $\nabla\mathbf{V}$, $W = \frac{(\nabla \mathbf{V}^t - \nabla \mathbf{V})}{2}$ is the antisymmetric part of $\nabla\mathbf{V}$, λ_1 and λ_2 are the relaxation and delay times respectively, μ is the dynamic viscosity, $\frac{d}{dt}$ is the material derivative, and $\breve{a}, \breve{b}$ and $\breve{c}$ are arbitrary material constants.

3. Formulation of the problem

Consider the steady and incompressible flow of Jeffrey fluid in symmetric channel. The surface is maintained at uniform constant temperature (Fig. 1).

The motion of an incompressible fluid is caused by sinusoidal wave trains propagating with constant speed c along the channel wall as given by the following:

$$h'(X', t') = a + b\sin\left(\frac{2\pi}{\lambda}(X' - ct')\right). \tag{8}$$

In above equation, b is the amplitude of the waves and $2a$ is the channel width, λ is the wavelength. X' and Y' are the rectangular Cartesian coordinates. The wall $Y = h'$ is kept at a temperature T_0. Introducing a wave frame (x', y') moves with velocity c away from the fixed frame (X', Y') with velocity c by the transformation

$$x' = X' - ct', \quad y' = Y', \quad u'(x', y') = U'(X', Y', t') - c, \quad v'(x', y') = V'(X', Y', t'), \tag{9}$$

where (u', v') are the velocity components in the wave frame (x', y'). The equations governing the fluid motion in the wave frame are as follows:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0, \tag{10}$$

$$\rho\left[u'\frac{\partial}{\partial x'} + v'\frac{\partial}{\partial y'}\right]u' = -\frac{\partial p'}{\partial x'} + \frac{\partial S'_{x'x'}}{\partial x'} + \frac{\partial S'_{x'y'}}{\partial y'} + \rho g \beta_T (T - T_0), \tag{11}$$

$$\rho\left[u'\frac{\partial}{\partial x'} + v'\frac{\partial}{\partial y'}\right]v' = -\frac{\partial p'}{\partial y'} + \frac{\partial S'_{y'x'}}{\partial x'} + \frac{\partial S'_{y'y'}}{\partial y'}, \tag{12}$$

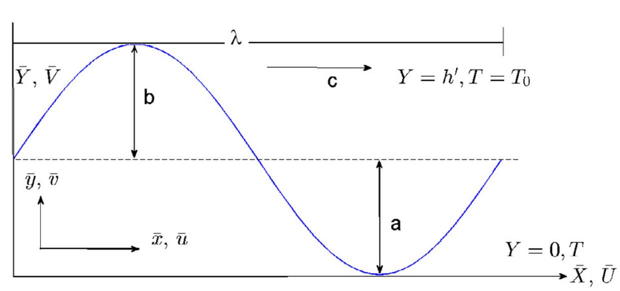

Figure 1 Flow geometry.