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Collapse and revival of entanglement of two-qubit in superconducting quantum dot lattice with magnetic flux and inhomogeneous gate voltage

Sujit Sarkar *

PoornaPrajna Institute of Scientific Research, 4 Sadashivanagar, Bangalore 5600 80, India

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ABSTRACT

We study the entanglement of a two-qubit system in a superconducting quantum dot (SQD) lattice in the presence of magnetic flux and gate voltage. The ground state is always in a maximally entangled Bell state for homogeneous gate voltage. In the presence of inhomogeneous gate voltage, the half-integer magnetic flux quantum, completely washes out the entanglement of the system at zero temperature. The entanglement is much higher for the Mott insulating phase. At finite temperature, collapse of entanglement occurs for wider region of magnetic flux.

1. Introduction

A very intense research activities have started in recent years involving an interaction between the subjects of quantum information science and quantum many body physics [1–5]. Quantum entanglement is the pivot of quantum information science, a truly unique feature of quantum mechanical systems, it has no classical analogue. Entanglement implies non-local correlations between particles in quantum many body systems. The quantum phase transition is one of the elegant phenomena of quantum many body system associate with different physical phenomena of that system. It has been observed recently that the quantum phase transition of the system is also associated with the quantum entanglement physics of the system and it has been observed that the entanglement of the quantum system extends over the macroscopic distances as ordinary correlation do. For example, let us consider a many body interacting quantum spin system. The ground state wave function changes qualitatively in a quantum phase transition. Therefore, one should be curious to see how the quantum entanglement changes as the transition point is traversed. The fidelity is an important concept to measure the quantum entanglement in the system. The fidelity typically drops in an abrupt manner at a quantum critical point indicating a dramatic change in the nature of the ground state wave function [6–11].

We have been motivated by the extensive studies of entanglement physics and interesting results in spin system [12–25]. We decided to apply this concept of study in the different disciplines of quantum condensed matter many-body system. Here we study the entanglement physics of two qubit in superconducting quantum dot lattice. Before we proceed further, we would like to discuss the basic aspects of a superconducting quantum dot lattice: Superconducting quantum dot lattice consists of array of superconducting grains. The superconducting grains are of nanoscale size, and different states of the superconducting grain are controlled by the ratio between charging energy and Josephson energy, while the average charge of the dot is controlled by the gate voltage [26–32] lattice. Superconducting circuits (qubit lattice) have attracted considerable interest in the recent years owing to the interesting physical properties. Superconducting circuits are macroscopic in size but have generic quantum properties such as quantized energy levels, superposition of states and entanglement. Superconducting circuits are generally hundreds of nanometer wide and contains trillions of electrons but they posses quantum nature. The quantum nature of this circuit is observable because they can be represented by a single degree of freedom. This most attractive feature of these superconducting circuits has opened a new area of fundamental science and the other part is the long term potential for quantum computing [5]. There are three types of superconducting qubits, namely, charge, flux and phase qubits. These qubits are operating based on two fundamental properties of superconductors such as flux quantization and Josephson tunneling [33–36].

One can consider the Cooper pair of SQD as a charged boson. The Physics of SQD can therefore be described in terms of interacting bosons. Bosonic physics is more interesting and hard to understand than the fermionic physics. At the same time quantum phase diagram of this SQD lattice is very rich with

* Tel.: +91 80 23617465.
E-mail address: sujit.tifr@gmail.com

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different quantum phases. Therefore the study of the entanglement physics for the SQD lattice for both zero and finite temperature is interesting in its own right [25–27].

It is well known from our previous studies that mesoscopic SQUID array can also be treated as the superconducting quantum dot lattice with modulated Josephson junction [25,26]. The authors of Ref. [29] have found the magnetic flux induced superconducting Coulomb blocked in mesoscopic SQUID array and also the magnetic flux induced superconductor–insulator quantum phase transition. Experimentally and also theoretically it reveals that the applied magnetic flux has an important effect in the SQD lattice system. We will see in due course of our study that the inhomogeneity of the gate voltage plays an important role in entanglement to disentanglement (product state) transition. Therefore we are motivated to study entanglement physics of SQD lattice in the presence of applied magnetic flux and inhomogeneity of gate voltage. Here we consider the two qubits of SQD lattice which is sufficient to predict entanglement to disentanglement transition of our system. At the same time our approach is completely analytical and there is no approximations to study this two qubits problem. We will see that this minimal model contains many important results. In the previous literature, there are few studies for two qubits for different physical systems [4,24,25]. The plan of the paper is as follows. We present the model Hamiltonian and entanglement physics in Section 2 of this paper. We present summary and conclusion in Section 3 of the paper.

2. Model Hamiltonian for inhomogeneous superconducting quantum dot lattice and the study of entanglement physics

2.1. Model Hamiltonian and ground state analysis

At first we write down the model Hamiltonian of SQD lattice system with Josephson couplings having on-site charging energies and inter-site interactions in the presence of gate voltage and external magnetic flux. We also consider the inhomogeneity in the applied gate voltage. The Hamiltonian is written as

$$H = H_{J1} + H_{EC0} + H_{EC1}. \tag{1}$$

We recast different parts of the Hamiltonian in quantum phase model as

$$H_{J1} = -E_{J1}\left|\cos\left(\pi\frac{\Phi}{\Phi_0}\right)\right|\sum_i \cos(\phi_{i+1}-\phi_i),$$

where ϕ_i and ϕ_{i+1} are quantal phase of the SQD at the point i and $i+1$ respectively, as

$$H_{EC0} = \frac{E_{C0}}{2}\sum_i\left(-i\frac{\partial}{\partial\phi_i}-\frac{N_i}{2}\right)^2,$$

where E_{C0} is the on-site charging energy. Now

$$H_{EC1} = E_{Z1}\sum_i n_i n_{i+1},$$

where E_{Z1} is the NN charging energies between the dots respectively. In the phase representation, $(-i\partial/\partial\phi_i)$ is the operator representing the number of Cooper pairs at the ith dot, and thus it takes only the integer values (n_i). Here, Hamiltonian H_{EC0} accounts for the influence of gate voltage $(eN \sim V_g)$, where eN is the average dot charge induced by the gate voltage. When the ratio $E_{J1}/E_{C0} \to 0$, the SQD array is in the insulating state having a gap of the width $\sim E_{C0}$, since it costs an energy $\sim E_{C0}$ to change the number of pairs at any dot. The exceptions are the discrete points at $N=(2n+1)$, where a dot with charge $2ne$ and $2(n+1)e$ has the same energy because the gate charge compensates the charges of extra Cooper pair in the dot. On this degeneracy point, a small amount of Josephson coupling leads the system to the superconducting state.

Here we recast our basic Hamiltonians in the spin language, where each site of the dot is either empty or singly occupied. During this process we follow Refs. [26,30]. Now

$$H_{J1} = -2E_{J1}\left|\cos\left(\pi\frac{\Phi}{\Phi_0}\right)\right|\sum_i (S_i^{\dagger}S_{i+1}^{-} + h.c.),$$

and

$$H_{EC0} = \frac{E_{C0}}{2}\sum_i (2S_i^Z - h)^2,$$

$$H_{EC0} = -2E_{C0}\sum_i h_i S_i^Z.$$

Here $h_i = (N_i - 2n - 1)/2$ allows the tuning of the system around the degeneracy point by means of gate voltage. We can tune the gate voltage in such a way that we can generate inhomogeneity in on-site charging energy. Without loss of generality we can also write the model Hamiltonian as

$$H_{EC0} = \sum_i (E_{C0}+\delta V_g)S_i^z + \sum_i (E_{C0}-\delta V_g)S_{i+1}^z. \tag{2}$$

where δV_g is the variation of gate voltage around the lattice sites. $H_{EC1} = E_{Z1}\sum_i S_i^z S_{i+1}^z$.

The total Hamiltonian of the system is

$$H = 2E_{J1}\left|\cos\left(\pi\frac{\Phi}{\Phi_0}\right)\right|\sum_i (S_i^{\dagger}S_{i+1}^{-} + h.c) + E_{Z1}\sum_i S_i^z S_{i+1}^z + \sum_i (E_{C0}+\delta V_g)S_i^z + \sum_i (E_{C0}-\delta V_g)S_{i+1}^z. \tag{3}$$

Now we consider the Hamiltonian for $N=2$ case. We would like to write the Hamiltonian in the standard basis, $|1,1\rangle$, $|1,0\rangle$, $|0,1\rangle$, $|0,0\rangle$

$$H = \begin{pmatrix} E_{Z1}+E_{C0} & 0 & 0 & 0 \\ 0 & -E_{Z1}+\delta V_g & B & 0 \\ 0 & B & -E_{Z1}-\delta V_g & 0 \\ 0 & 0 & 0 & E_{Z1}-E_{C0} \end{pmatrix}$$

$B = 2E_{J1}|\cos(\pi\Phi/\Phi_0)|$. The eigenstates of these two Hamiltonian sites are $|\psi_1\rangle = |0,0\rangle$, $|\psi_1\rangle = |1,1\rangle$, $|\psi_3\rangle = \sqrt{1/(1-c_1^2)}(c_1|1,0\rangle + |0,1\rangle)$, $|\psi_4\rangle = \sqrt{1/(1+c_2^2)}(c_2|1,0\rangle - |0,1\rangle)$. Where $c_1 = (\delta V_g - A)/B$, $A = \sqrt{\delta V_g^2 + 4E_J^2|\cos(\pi\Phi/\Phi_0)|^2}$, $c_2 = (\delta V_g + A)/B$. $E_1 = \frac{1}{2}(2E_{Z1} - 2E_{C0})$, $E_2 = \frac{1}{2}(2E_{Z1} + 2E_{C0})$, $E_3 = -E_{Z1} - A$ $E_4 = -E_{Z1} + A$. If we consider the homogeneous system, i.e., there is no variation of gate voltage over the lattice sites. The two states $|\psi_3\rangle$ and $|\psi_4\rangle$ are the maximally entangled Bell states, i.e., $(1/\sqrt{2})(|0,1\rangle - |1,0\rangle)$, $(1/\sqrt{2})(|0,1\rangle + |1,0\rangle)$. As we see from our analytical expression that ground state depends on the value of E_{C0}, E_{Z1} and A. Ground state is in the disentangle state (product state) when the ground state energy is either E_1 or E_2, otherwise the system is in the entangle state. Thus for this superconducting quantum dot lattice system there is a transition between the disentangle state to entangle state due to the variation of the system parameters. We will see in due course of our study that the magnetic flux plays an important role in the transition between the disentangled state to the entangle state.

2.2. Entanglement study for zero and finite temperature

Now we calculate the thermal entanglement of two arbitrary qubits $(N=2)$ of the superconducting quantum dot lattice. The