Alexandria Engineering Journal (2013) 52, 801–805

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

A new type of shooting method for nonlinear boundary value problems

Muhammad Ahsan *, Sarah Farrukh

School of Chemical and Materials Engineering, National University of Sciences and Technology, H-12, Islamabad 44000, Pakistan

Received 11 April 2013; revised 16 July 2013; accepted 18 July 2013
Available online 12 August 2013

KEYWORDS

Boundary value problem;
New shooting method;
Nonlinear boundary value problems;
Initial value problems;
Interpolation

Abstract In this article we introduce a new type of iterative method for initial value problems (IVPs). We enhance this method by using shooting techniques and interpolation for the boundary value problems. Our method is more accurate and applicable than built in methods used in different software packages. We solved several examples for initial value problems and linear and non-linear boundary value problems and compared results to those obtained using MATLAB.

1. Introduction

Many techniques for solving boundary value problems were discussed and presented by many researchers. The common technique for solving boundary value problems is shooting method. In shooting method the boundary value problem (BVP) is reduced to the solution of an initial value problem (IVP), by assuming initial values that would have been given if the ordinary differential equation were an initial value problem. The boundary value calculated is then matched with the real boundary value. Using trial and error or some scientific approach, one attempts to get as close to the boundary value as possible [1,2]. The concept of multiple shooting was first suggested by Morrison et al. [3] later promoted by Keller [4] who developed and analyzed both a simple shooting method (SSM) and a multiple shooting method (MSM). A more latest version of a multiple shooting method, MUSN, was developed by Mattheij and Staarink [5]. Lie-group shooting method was proposed by Liu [6–8].

In this article, we develop an iterative formula for the initial value problem (IVP) and convert it to boundary value problem by shooting technique and interpolation. The mostly used shooting technique in different software depends on Newton Raphson method, which fails to predict results when the first derivative of function is zero or undefined .Thus, technique fails in many problems, while we used interpolation to approximate the guess, due to this we can solve several problems using shooting method that could not be solved. This article is organized as follows. Section 1 is an introduction and Section 2 introduces a new approximating formula for the IVP. Section 3 describes the shooting technique and in Section 4 some nonlinear problems are solved using the proposed shooting method. Section 5 concludes the paper.

* Corresponding author. Tel.: +92 3336057937.
E-mail address: ahsan@scme.nust.edu.pk (M. Ahsan).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

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http://dx.doi.org/10.1016/j.aej.2013.07.001

2. New approximating formula for initial value problem

Let us consider the nonlinear initial value problem

$$y' = f(x,y), \quad y(x_o) = y_o$$

We consider the Taylor series expansion of $y(x+h)$

$$y(x+h) = y(x) + hy'(x) + \frac{1}{2}h^2y''(x) + \frac{1}{3!}h^3y'''(x) + O(h^4) \tag{1}$$

where the given nonlinear part of the initial value problem is $y'(x) = f(x,y)$

Now, we can further approximate for second and third derivatives,

$$y''(x) = f_x(x,y) + f_y(x,y)y'(x)$$

$$y''(x) = f_x(x,y) + f_y(x,y) \cdot f(x,y)$$

$$y'''(x) = f_{xx}(x,y) + f_{yx}(x,y)y'(x) + f_{yy}(x,y)(y'(x))^2$$

$$y'''(x) = f_{xx}(x,y) + f_{yx}(x,y) \cdot f(x,y) + f_{yy}(x,y)f^2(x,y)$$

So putting all derivatives approximations in (1)

$$y(x+h) = y(x) + hf(x,y) + \frac{1}{2}h^2f_x(x,y) + \frac{1}{2}h^2f_y(x,y)f(x,y) + \frac{1}{3!}h^3f_{xx}(x,y) + \frac{1}{3!}h^3f_{yx}(x,y)f(x,y) + \frac{1}{3!}h^3f_{yy}(x,y)f^2(x,y) \tag{2}$$

Consider the equation proposed for our iterative formula;

$$y(x+h) = y(x) + Ahf_0 + Bhf_1 + Chf_2 \tag{3}$$

where

$$f_0 = f(x,y) \tag{4}$$

$$f_1 = f(x+Ph, y+Qhf_0) \tag{5}$$

$$f_2 = f(x+Rh, y+Shf_1) \tag{6}$$

$$f_1 = f(x,y) + Phf_x(x,y) + Qhf_y(x,y)f(x,y) \tag{7}$$

$$f_2 = f(x,y) + Rhf_x(x,y) + Shf_y(x,)f(x,y) + \frac{S^2h^2}{2}f_{yy} + SRh^2f_x(x,y) + R^2h^2f_{xx}(x,y) \tag{8}$$

Our aim is to find the values of the constants A, B, C, P, Q, R and S, putting (4)–(8) in (3)

$$y(x+h) = y(x) + Ahf(x,y) + Bhf(x,y) + BPh^2f_x(x,y) + BQh^2f_y(x,y)f(x,y) + Chf(x,y) + CRh^2f_x(x,y) + CSh^2f_y(x,y)f(x,y) + CS^2\frac{h^3}{2}f_{yy}(x,y) + CSRh^3f_x(x,y)f_y(x,y) + CR^2h^3f_{xx}(x,y)$$

which implies

$$y(x+h) = y(x) + hf(x,y)[A+B+C] + h^2f_x(x,y)[BP + CR] + h^2f_y(x,y)[BQ+CS] + CS^2 \times \frac{h^3}{2}f_{yy}(x,y) + CSRh^3f_xf_y + CR^2\frac{h^3}{2}f_{xx} \tag{9}$$

Comparing (2) and (9)

$$A + B + C = 1$$

Let

$$A = B = C = 1/3$$

$$BP + CR = 1/2$$

$$BQ + CS = 1/2$$

$$CR^2 = 1/3$$

$$1/3(P+R) = 1/2 \Rightarrow P + R = 3/2 \Rightarrow P = 1/2$$

$$1/3(Q+S) = 1/2 \Rightarrow Q + S = 3/2$$

$$R^2 = 1 \Rightarrow R = 1$$

Solving above equations for the constants and putting values in Eqs. (4)–(8) for solving (3), we have our required formula as

$$f_0 = f(x,y)$$

$$f_1 = \left(x + \frac{h}{2}, y + hf_0\right)$$

$$f_2 = \left(x + h, y + \frac{h}{2}f_1\right)$$

$$y = y(x) + \frac{h}{3}(f_0 + f_1 + f_2) \tag{10}$$

3. Shooting method

Let us consider the two point non-linear boundary value problem

$$y'' = f(x,y,y'), \quad y(a) = \alpha, \quad y(b) = \beta \tag{A}$$

Our aim is to convert the above boundary value problem into initial value problem. In this case, (A) equation can be written as for IVP.

$$y'' = f(x,y,y'), \quad y(a) = \alpha, \quad y'(a) = Unknown = s\ (Say) \tag{A}$$

Now, the goal is to find s for which the solution satisfies the second boundary condition

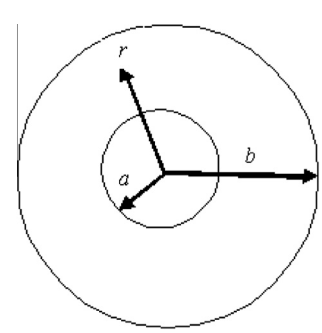

Figure 1 Cross sectional geometry of a pressure vessel.