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A theoretical study of nonlinear optical absorption and refractive index changes with the three-dimensional ring-shaped pseudoharmonic potential

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ABSTRACT

Linear and nonlinear optical absorption coefficients and refractive index changes with the three-dimensional ring-shaped pseudoharmonic potential are theoretically investigated. The wave functions and the energy levels are obtained by using separation of variables. It is found that the optical absorption coefficients and the refractive index changes are strongly affected not only by the chemical potential V_0 and the zero point of the pseudoharmonic potential r_0, but also by the parameter β of the ring-shaped potential.

1. Introduction

The rapid progress in nanofabrication techniques such as molecular beam epitaxy and metal–organic chemical vapor deposition have made it possible to fabricate low-dimensional semiconductor quantum systems, which will help to obtain obvious nonlinear optical effects such as optical rectification, optical absorption, refractive index change, second harmonic generation, and third harmonic generation experimentally. The nonlinear optical properties have the potential for application in many optoelectronic devices such as high-speed electro-optical modulators, far infrared photodetectors, semiconductor optical amplifiers and so on.

Recently, a number of researchers have done theoretical researches on nonlinear optical properties [1–33,36,38–44]. These researches show that compared to in bulk materials, the strong quantum confinement effects in low-dimensional semiconductor quantum systems will greatly improve nonlinear optical effects. In the researches above, changing the shapes and sizes of quantum systems is often used to do researches on nonlinear optical effects. For example, cubic [3,4], spherical [5–8], disk-shaped [9–12], cylindrical [13–15], ring [16] quantum dots are studied by some researchers. Besides, there are many physical factors that have great influences on nonlinear optical effects. Therefore, some authors do some researches by adding the number of electrons such as two electrons [9,17,18] and consider polaron [13,19–21] and excitonic [22–24] effects. Moreover, the application of an electric field [14,16,18,25], a magnetic field [12,13,26], impurity [10,16,25], spin–orbit interaction [4,27] and external perturbation like hydrostatic pressure [14,16], temperature [14,16] and intense laser field [14,28,29], exhibits novel nonlinear optical phenomena. Besides, changing the confinement potential in quantum structures is a very useful method. For example, the parabolic potential [1,2], the Woods–Saxon potential [31], the hyperbolic potential [32], the exponential potential [33], and the linear potential [13] are used. In our paper, we make use of a kind of combination potential which includes the pseudoharmonic potential [34] and the ring-shaped potential [35] to study the nonlinear optical absorption and refractive index changes. The ring-shaped potential was studied by Liu and Guo [36] with the fact that the ring-shaped potential has obvious influences on the linear and the nonlinear optical absorption coefficients and refractive index changes. The pseudoharmonic potential $V(r)=V_0((r/r_0)-(r_0/r))^2$ was earlier investigated by Cetin, Bogachek and Landman [34,37]. Their works show how to fabricate the pseudoharmonic potential and give some basic properties about the system. Based on their works, much work about the pseudoharmonic potential has been done by some other researchers [38–44]. For example, Rezaei et al. investigated intersubband optical absorption coefficient changes and refractive index changes optical rectification coefficient in the pseudoharmonic potential system [38,39]. Xie gave a study of optical absorption and refractive index of a donor impurity in the three-dimensional pseudoharmonic potential system [40]. Besides, he also discussed electron Raman scattering in the two-dimensional pseudoharmonic potential system [41]. Li et al.

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investigated polaron effects on the optical absorption coefficients and refractive index changes in the two-dimensional quantum pseudodot system [42]. Yu studied third-harmonic generation in the two-dimensional pseudoharmonic potential system with an applied magnetic field [43]. Liang and Xie discussed optical properties in the two-dimensional pseudoharmonic potential quantum ring: confinement potential and Aharonov–Bohm effect [44]. All these researches show that the nonlinear optical properties in pseudoharmonic potential indeed have novel effects. Therefore, it is necessary to give a detailed study of the scheme of combination potential above.

This paper is organized as follows. In Section 2, we obtain the eigenfunctions and the energy eigenvalues. The analytical expressions for the linear and nonlinear optical absorption coefficients and refractive index changes are presented. In Section 3, the numerical results and discussions are performed. Finally, a brief conclusion is made in Section 4.

2. Theory

Let us consider an electron confined in the system. Within the framework of effective mass approximation, the Hamiltonian of the system, is given by

$$H=-\frac{\hbar^2}{2m^*}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]+V(r), \tag{1}$$

where m^* is the electron effective mass, and $V(r)$ is the potential which includes the pseudoharmonic potential [34] and the ring-shaped potential [35] as follows:

$$V(r)=V_0\left(\frac{r}{r_0}-\frac{r_0}{r}\right)^2+\frac{\hbar^2}{2m^*}\frac{\beta}{r^2\sin^2\theta}, \tag{2}$$

where V_0 is the chemical potential, r_0 is the zero point of the pseudoharmonic potential, and β is the dimensionless parameter.

The schrödinger equation in spherical coordinate system has the following form:

$$H\psi(r,\theta,\phi)=E\psi(r,\theta,\phi). \tag{3}$$

The wave function can be written as

$$\psi(r,\theta,\phi)=R(r)P(\theta)\Phi(\phi). \tag{4}$$

Substituting Eq. (4) into Eq. (2) allows us to obtain the following three equations:

$$(1-x^2)\frac{d^2P(x)}{dx^2}-2x\frac{dP(x)}{dx}+\left[\lambda-\frac{\xi^2}{1-x^2}\right]P(x)=0, \tag{5}$$

with

$$\xi=\sqrt{\beta+m^2},\quad \lambda=l'(l'+1),\quad x=\cos(\theta), \tag{6}$$

$$\frac{d^2\Phi(\phi)}{d\phi^2}+m^2\Phi=0, \tag{7}$$

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\left[a^2-\frac{L(L+1)}{r^2}-b^2r^2\right]R=0, \tag{8}$$

with

$$L=\frac{1}{2}\left[\sqrt{1+4\left[\frac{2m^*V_0r_0^2}{\hbar^2}+(\sqrt{\beta+m^2}+h)(\sqrt{\beta+m^2}+h+1)\right]}-1\right], \tag{9}$$

$$a^2=\frac{2m^*(E+2V_0)}{\hbar^2},\quad b^2=\frac{2m^*V_0}{\hbar^2r_0^2}. \tag{10}$$

The normalized solution of Eq. (5) can be obtained as

$$P_{l'\xi}(\theta)=\sqrt{\frac{(2l'+1)}{2}\frac{(l'-\xi)!}{\Gamma(l'+\xi+1)}}(\sin\theta)^{\xi}\sum_{\kappa=0}^{[(l'-\xi)/2]}A(\cos\theta)^{l'-\xi-2\kappa}, \tag{11}$$

with

$$A=\frac{(-1)^{\kappa}\Gamma(2l'-2\kappa+1)}{2^{l'}\kappa!(l'-\xi-2\kappa)!\Gamma(l'-\kappa+1)},\quad l'-\xi=h,h=0,1,2,\ldots \tag{12}$$

where the parameters l' and ξ do not need to be integer, and just their difference must be integer. The normalized solution of Eq. (7) can be written as

$$\Phi(\phi)=\frac{1}{\sqrt{2\pi}}e^{im\phi},\quad m=0,\pm1,\pm2,\ldots \tag{13}$$

The normalized wave function of Eq. (8) can be obtained as

$$R_n(\rho)=N_n\rho^{L/2}e^{-\rho/2}L_n^{L+1/2}(\rho), \tag{14}$$

where $\rho=br^2$ and $N_n=b^{3/4}\sqrt{(2n!)/((L+n+1/2)!)}$. $L_n^{L+1/2}(\rho)$ is associated with the Laguerre polynomials. The energy eigenvalue of Eq. (8) can be obtained as

$$E_n=(2n+L+3/2)\hbar\sqrt{\frac{2V_0}{m^*r_0^2}}-2V_0. \tag{15}$$

As is shown above, the wave functions and the energy eigenvalue of the total system have been obtained.

By using the iterative method and the compact-density-matrix approach, the linear and the nonlinear optical absorption coefficients can be found as [13,38,40,45,46]

$$\alpha^{(1)}(\omega)=\omega\sqrt{\frac{\mu}{\epsilon_R}}\frac{|M_{21}|^2\sigma_{\upsilon}\hbar\Gamma_0}{(E_{21}-\hbar\omega)^2+(\hbar\Gamma_0)^2}, \tag{16}$$

$$\alpha^{(3)}(\omega,I)=-\omega\sqrt{\frac{\mu}{\varepsilon_R}}\left(\frac{I}{2\varepsilon_0n_rc}\right)\frac{|M_{21}|^2\sigma_{\upsilon}\hbar\Gamma_0}{[(E_{21}-\hbar\omega)^2+(\hbar\Gamma_0)^2]^2}\times\left\{4|M_{21}|^2-\frac{|M_{22}-M_{11}|^2[3E_{21}^2-4E_{21}\hbar\omega+\hbar^2(\omega^2-\Gamma_0^2)]}{E_{21}^2+(\hbar\Gamma_0)^2}\right\}. \tag{17}$$

The total optical absorption coefficients can be written as

$$\alpha(\omega,I)=\alpha^{(1)}(\omega)+\alpha^{(3)}(\omega,I). \tag{18}$$

Based on the same method, the linear and the third-order nonlinear refractive index changes can be expressed as [13,38,40,45,46]

$$\frac{\Delta n^{(1)}(\omega)}{n_r}=\frac{\sigma_{\upsilon}|M_{21}|^2}{2n_r^2\varepsilon_0}\frac{E_{21}-\hbar\omega}{(E_{21}-\hbar\omega)^2+(\hbar\Gamma_0)^2}, \tag{19}$$

$$\frac{\Delta n^{(3)}(\omega,I)}{n_r}=-\frac{\sigma_{\upsilon}|M_{21}|^2}{4n_r^3\varepsilon_0}\frac{\mu cI}{[(E_{21}-\hbar\omega)^2+(\hbar\Gamma_0)^2]^2}\left\{4(E_{21}-\hbar\omega)|M_{21}|^2+\frac{(M_{22}-M_{11})^2}{(E_{21})^2+(\hbar\Gamma_0)^2}\times\{(\hbar\Gamma_0)^2(2E_{21}-\hbar\omega)-(E_{21}-\hbar\omega)[E_{21}(E_{21}-\hbar\omega)-(\hbar\Gamma_0)^2]\}\right\}. \tag{20}$$

The total refractive index changes can be written as

$$\frac{\Delta n(\omega,I)}{n_r}=\frac{\Delta n^{(1)}(\omega)}{n_r}+\frac{\Delta n^{(3)}(\omega,I)}{n_r}, \tag{21}$$

where σ_{υ} is the electron density, $E_{21}=E_2-E_1$ the energy interval of two different electronic states, $M_{12}=\langle\psi_1|r\cos\theta|\psi_2\rangle$ the dipole transition matrix element, n_r the refractive index, μ the permeability, ε_R the real part of the permittivity, ω the incident photon frequency, I the incident optical intensity, and Γ_0 the phenomenological relaxation rate.