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A time-dependent stop operator for modeling a class of singular hysteresis loops in a piezoceramic actuator

Mohammad Al Janaideh

Department of Mechatronics Engineering, The University of Jordan, 11942 Amman, Jordan

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ABSTRACT

We present a time-dependent stop operator-based Prandtl–Ishlinskii model to characterize singular hysteresis loops in a piezoceramic actuator. The model is constructed based on the time-dependent threshold. The inverse time-dependent stop operator-based Prandtl–Ishlinskii model is obtained analytically and it can be applied as a feedforward compensator to compensate for singular hysteresis loops in a class of smart-material-based actuators. The objective of this study is to present an invertible Prandtl–Ishlinskii model that can be applied as a feedforward compensator to compensate for singular hysteresis loops without inserting a feedback control system.

1. Introduction

Smart-material-based actuators such as piezoceramic and magnetostrictive actuators are becoming increasingly popular for high frequency micro- and nano-positioning applications because of a number of advantages which include nanometer resolution and fast response [1–6]. However, under high excitation frequencies, a number of smart-material-based actuators exhibit singular hysteresis loops between the applied input voltage or current and measured output displacement, see for example [7–15]. These nonlinearities cause oscillations, poor tracking performance, and potential instabilities [1].

The Prandtl–Ishlinskii model is an attractive choice for generating input–output hysteresis loops of piezoceramic and magnetostrictive actuators since the model can be implemented with a few hysteresis operators, see for example [12–14]. The model can characterize time-dependent hysteresis nonlinearities and creep effects [7,10]. Furthermore, The inverse Prandtl–Ishlinskii model can be obtained analytically [16,18]. Consequently, the inverse model can be applied as a feedforward compensator to compensate for hysteresis effects in real-time systems [13,10].

Different hysteresis models have been proposed to model the hysteresis nonlinearities between the input voltage and the output displacement in piezoceramic actuators, see for example [2]. However, hysteresis modeling in most studies have been carried out under low excitation frequencies, where the piezoceramic actuators show nonsingular hysteresis loops. In order to design a suitable control system compensates for the these nonlinearities, it is essential to characterize nonsingular hysteresis loops. A number of studies have applied a rate-independent hysteresis model as well as a transfer function describes the dynamics of the piezoceramic actuator to characterize the hysteresis loops at different excitation frequencies, see for example [8]. In this paper, we propose a time-dependent Prandtl–Ishlinskii model to characterize singular hysteresis loops.

2. The time-dependent stop operator-based Prandtl–Ishlinskii model

The stop operator has been proposed to characterize the elastic–plastic behavior in continuum mechanics [19]. In this section, the time-dependent stop operator-based Prandtl–Ishlinskii model is presented.

2.1. The model

We deal with the space $AC(0,T)$ of real absolutely continuous functions defined on the interval $[0,T]$. For an input $u(t) \in AC(0,T)$, the output of the time-dependent stop operator is denoted as

$$w(t) = \Phi_{r(t)}[u,x](t), \tag{1}$$

where x is an initial condition. For inputs and thresholds that are piecewise linear, that is, linear in each interval of a partition $0 = t_0 < t_1 < \cdots < t_l = T$, the output of the time-dependent play operator expressed as

$$z(t) = \Gamma_{r(t)}[u,x](t), \tag{2}$$

where

$$\Gamma_{r(t)}[u,x](t) = \max(u(t) - r(t), \min(u(t) + r(t), z(t_{i-1}))). \tag{3}$$

E-mail address: aljanaideh@gmail.com

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We define the time-dependent stop operator-based on the complementary property [19] as

$$\Phi_{r(t)}[u,x](t) = u(t) - \Gamma_{r(t)}[u,x](t). \quad (4)$$

Let $n \in \mathbb{N}$, the output of the time-dependent stop operator-based Prandtl–Ishlinskii model formulated for $i = 1,2,\ldots,n$ as

$$\Pi[u](t) = a_0 u(t) + \sum_{i=1}^{n} a_i \Phi_{r_i(t)}[u,x_i](t) \quad (5)$$

where a_0 is a positive constant and a_i are negative constants. The dynamic thresholds $r_i(t)$ are defined for $t \in [0,T]$ as

$$0 \le r_1(t) \le r_2(t) \le \cdots \le r_n(t). \quad (6)$$

To characterize the singular hysteresis loops in smart actuators, we use the following time-dependent threshold function

$$r_i(t) = \alpha_i + \beta_i |\dot{u}(t)|, \quad (7)$$

where $0 \le \beta_i \ll 1$

$$\alpha_{i+1} - \alpha_i \ge \sigma \quad (8)$$

and

$$\beta_{i+1} - \beta_i \ge \eta, \quad (9)$$

where σ and η are positive constants and $\alpha_i \gg \beta_i$.

It is important to note that the time-dependent stop operator-based Prandtl–Ishlinskii model is reduced to the stop operator-based Prandtl–Ishlinskii model with $\beta_i = 0$ and $r_i(t) = \alpha_i$.

2.2. Example

An input signal of the form $u(t) = 10 \sin(2\pi f t)$, where f is the excitation frequency. The following dynamic threshold is considered:

$$r(t) = \alpha + \beta |\dot{u}(t)|. \quad (10)$$

The simulation results with f=50 Hz and 100 Hz are shown in Fig. 1. It can be concluded that the time-dependent stop operator constructed with a suitable time-dependent threshold function $r(t)$ can exhibit singular hysteresis loops. Fig. 1(d) shows input–output relationship of the stop operator presented in Ref. [19].

3. Hysteresis modeling

In this section, the stop operator-based Prandtl–Ishlinskii model (5) is applied to characterize singular time-dependent hysteresis loops of a piezoceramic actuator.

3.1. Experimental results

The experiments were performed on a piezoceramic actuator (P-753.31C) from Physik Instrumente Company. A capacitive sensor is used to measure the output displacement of the actuator. The actuator displacement response signal was acquired by a DSpace DS1104 controller board. The measurements with the piezoceramic actuator were performed under a harmonic input of $u(t) = 40 \cos(2f\pi t)$ V at three excitation frequencies (1 Hz, 200 Hz, and 550 Hz). The input voltage and output displacement signals were acquired at a sampling frequency of 10 kHz. Fig. 2 shows the

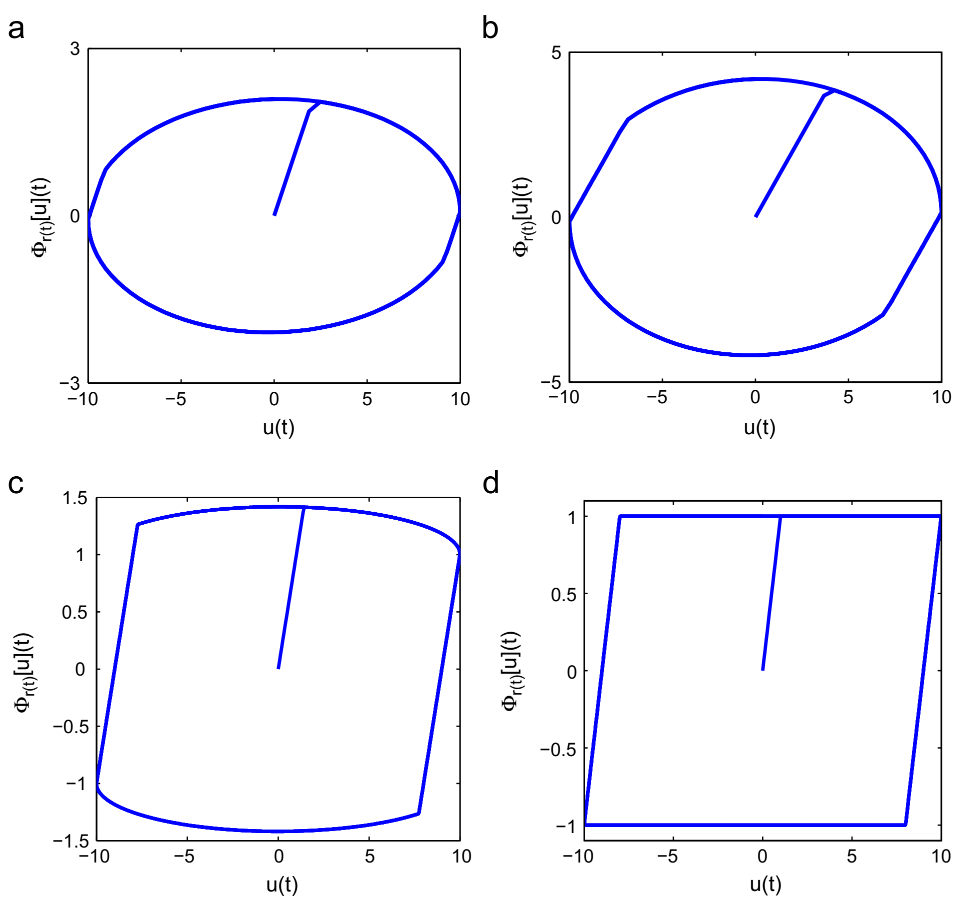

Fig. 1. The input–output characteristics of a time-dependent stop operator at: (a) 50 Hz with $\alpha = 1$ and $\beta = \frac{1}{1500}$, (b) 100 Hz with $\alpha = 1$ and $\beta = \frac{1}{1500}$, (c) 100 Hz with $\alpha = 1$ and $\beta = \frac{1}{15,000}$, and (d) 100 Hz with $\alpha = 1$ and $\beta = 0$.