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# Magnetohydrodynamic mixed convective slip flow over an inclined porous plate with viscous dissipation and Joule heating

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**Abstract** The combined effects of viscous dissipation and Joule heating on the momentum and thermal transport for the magnetohydrodynamic flow past an inclined plate in both aiding and opposing buoyancy situations have been carried out. The governing non-linear partial differential equations are transformed into a system of coupled non-linear ordinary differential equations using similarity transformations and then solved numerically using the Runge–Kutta fourth order method with shooting technique. Numerical results are obtained for the fluid velocity, temperature as well as the shear stress and the rate of heat transfer at the plate. The results show that there are significant effects of pertinent parameters on the flow fields.


## 1. Introduction

Magnetohydrodynamic (MHD) mixed convective flows or combined free and forced convection past a flat plate has been widely studied from both theoretical and experimental standpoints over the past a few decades. MHD mixed convective flows occur in many technological and industrial applications, e.g. solar receivers exposed to wind currents, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity environment, lubrication purposes, drying technologies, flows in the ocean and in the atmosphere [1,2]. Depending on the forced flow direction, the buoyancy forces may aid (aiding or assisting mixed convection) or oppose (opposing mixed convection) the forced flow, causing an increase or decrease in heat transfer rate [3]. The problem of mixed convection resulting from the flow over a heated vertical plate is of considerable theoretical and practical interest. A detailed review of the subject, including exhaustive lists of references, can be found in the books by Bejan [4], Pop and Ingham [5], Jaluria [6] and Chen and Armaly [7]. References [8–17] are some examples of the recent relevant studies existing in the literature. Mukhopadhyay et al. [18] have presented the MHD combined convective flow past a stretching surface. The mixed convection of a viscous dissipating fluid about a vertical flat plate has been studied by Aydin and Kaya [19].

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The deviation from interfacial thermodynamic equilibrium will lead to a flow regime where the conventional no-slip wall condition is not valid. According to the value of Knudsen number, the flows can be classified into three categories: continuum flow ($Kn < 0.01$), slip flow ($0.01 \leqslant Kn \leqslant 0.1$) and transitional flow ($0.1 < Kn < 10$) [20]. As the flow deviates away from the continuum limit, the conventional no-slip wall boundary condition fails to accurately model the surface interaction between the fluid and the wall boundary due to the low collision frequency. Slip models have been proposed to ameliorate the prediction of the non-continuum phenomenon near wall boundaries within the framework of the continuum assumption. For large values of $Kn(> 10)$, the Navier–Stokes equations are not applicable and the kinetic theory of gases must be employed. In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle has a finite tangential velocity; it slips along the surface. The flow is called slip-flow and this effect cannot be neglected. Cao and Baker [21] have illustrated the slip effects on a mixed convective flow and heat transfer from a vertical plate. Aziz [22] has presented the hydrodynamic and thermal slip boundary layer flow over a flat plate with constant heat flux boundary condition. The combined effects of Joule heating and viscous dissipation on a magnetohydrodynamic free convective flow past a permeable stretching surface with radiative heat transfer have been determined by Chen [23]. Mukhopadhyay [24] has illustrated the slip effects on a unsteady mixed convective flow and heat transfer past a porous stretching surface. Bhattacharyya et al. [25] have investigated an MHD boundary layer slip flow and heat transfer over a flat plate. Rohni et al. [26] have studied an unsteady mixed convective boundary layer slip flow near the stagnation point on a vertical permeable surface embedded in a porous medium. Bhattacharyya et al. [27] have presented a mixed convective boundary layer slip flow over a vertical plate. The unsteady mixed convective flow from a moving vertical plate in a parallel free stream has been studied by Patil et al. [28].

Ellahi et al. [29] have presented a non-Newtonian MHD fluid flow with slip boundary conditions in porous space. A magnetohydrodynamic peristaltic flow of a Jeffrey fluid in eccentric cylinders has been investigated by Nadeem et al. [30]. The effects of temperature dependent viscosity on an MHD flow of non-Newtonian nanofluid in a pipe have been examined by Ellahi [31]. Zeeshan and Ellahi [32] have presented an MHD slip flow of non-Newtonian fluid in a porous space. Sheikholeslami et al. [33] have studied the Cu-water magneto-nanofluid flow and heat transfer. Ellahi et al. [34] have examined the effects of heat transfer and nonlinear slip on the steady Couette flow. Ellahi [35] has presented the magnetohydrodynamic peristaltic flow of Jeffrey fluid in a rectangular duct through a porous medium. Sheikholeslami et al. [36] have reported the CuO-water nanofluid flow and convective heat transfer considering Lorentz forces. Khana et al. [37] have investigated the effects of heat transfer on a peristaltic motion of Oldroyd fluid in the presence of inclined magnetic field. Akbar et al. [38] have investigated the influence of heat generation and heat flux in peristalsis with interaction of nanoparticles. Sheikholeslami et al. [39] have studied the natural convection of a nanofluid in an enclosure with elliptic inner cylinder. A mixed convective boundary layer flow over a vertical slender cylinder has been presented by Ellahi et al. [40].

The object of this paper was to investigate the combined effects of viscous dissipation and Joule heating on an MHD mixed convective flow past an inclined porous plate. The viscous and Joule dissipation effects are taken into consideration. The governing equations describing the problem are transformed into a non-linear ordinary differential equations by using similarity transformation. The transformed ordinary differential equations were solved numerically using fourth order Runge–Kutta method with the shooting technique. The effects of pertinent parameters on the fluid velocity and temperature have been shown graphically.

## 2. Mathematical formulation

Consider a mixed convective flow of a viscous incompressible electrically conducting fluid past a porous plate which is inclined from the vertical with an acute angle $\gamma$ measured in the clockwise direction and situated in an otherwise quiescent ambient fluid at temperature $T_\infty$. Choose a Cartesian coordinates system with $x$-axis along the plate and the $y$-axis is measured normal to the sheet in the outward direction toward the fluid (see Fig. 1(a)). A transverse magnetic field of strength $B$ is applied normal to the plate. The plate coincides with the plane $y = 0$ and the flow being confined to $y > 0$. It is assumed that the variation of fluid properties is taken to be negligible except for the essential density variation appearing in the gravitational body force.

Ohm's law is Cowling [41]

$$\vec{J} = \sigma\left(\vec{E} + \vec{q} \times \vec{B}\right), \tag{1}$$

where $\vec{q}$, $\vec{B}$, $\vec{E}$, $\vec{J}$ and $\sigma$ are respectively the velocity vector, the magnetic field vector, the electric field vector, the current density vector and the electrical conductivity. It is assumed that the magnetic Reynolds number is very small, so that induced magnetic field can be neglected [41]. This assumption is justified since the magnetic Reynolds number is generally very small for metallic liquid or partially ionized fluid. Liquid metals can be used in a range of applications because they are nonflammable, nontoxic and environmentally safe. That is why, liquid metals have number of technical applications

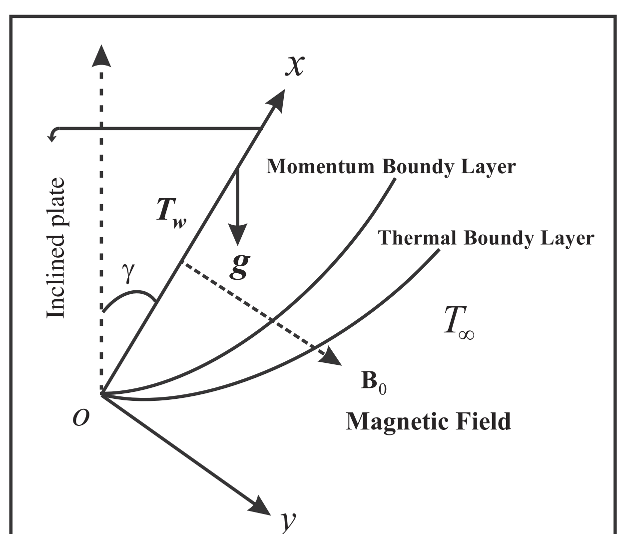


**Figure 1(a)** Geometry of the problem.