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ORIGINAL ARTICLE

Traveling wave solutions of some nonlinear evolution equations

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Abstract In this work, the modified simple equation (MSE) method is used to find exact traveling wave solutions to nonlinear evolution equations (NLEEs) in mathematical physics. To do so we have used the nonlinear Klein–Gordon (nKG) equation and the (2 + 1)-dimensional modified Zakharov–Kuznetsov (mZK) equation. Two classes of exact explicit solutions-hyperbolic and trigonometric solutions of the associated NLEEs are characterized with some free parameters. It is shown that the method provides a powerful mathematical tool for solving NLEEs in mathematical physics and engineering fields.

1. Introduction

Nowadays NLEEs i.e., partial differential equations with time derivatives, have become a useful tool for describing the natural phenomena of science and engineering. The investigation of traveling wave solutions of NLEEs plays a significant role in the study of nonlinear physical phenomena. The study of traveling wave solutions of NLEEs plays an important role to look into the internal mechanism of complicated physical phenomena. Most of the physical phenomena such as, fluid mechanics, quantum mechanics, electricity, plasma physics, chemical kinematics, propagation of shallow water waves, and optical fibers are modeled by nonlinear evolution equation, and the appearance of solitary wave solutions in nature is somewhat frequent. But, the nonlinear processes are one of the major challenges and not easy to control because the nonlinear characteristic of the system abruptly changes due to some small changes of valid parameters including time. Thus the issue becomes more complicated and hence ultimate solution is needed. Therefore, the study of exact solutions of NLEEs plays a vital role to understand the physical mechanism of nonlinear phenomena. Advanced nonlinear techniques are significant to solve inherent nonlinear problems, particularly those involved in dynamical systems and related areas. In recent years, there become significant improvements in finding the exact solutions of NLEEs. Many effective and powerful methods have been established and improved [25,27], such as, the Hirota's bilinear transformation method [13,14], the tanh-function method [16,20,26], the (G'/G)-expansion method [3–6,21], the Exp-function method [12,7,19,18], the homogeneous balance

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method [24,31], the F-expansion method [32], the Adomian decomposition method [2], the homotopy perturbation method [17], the extended tanh-method [1,10], the auxiliary equation method [22], the Jacobi elliptic function method [8], Weierstrass elliptic function method [15], modified Exp-function method [11], and the modified simple equation method [28–30].

Recently, Jawad et al. [28] proposed the MSE method to solve NLEEs arising in mathematical physics. Right after this pioneer work, this method became popular among the research community, and many studies refining the initial idea have been published [29,30].

The objective of this article was to look for new use relating to the MSE method for solving the nKG equation and mZK equation and demonstrates the advantage and straightforwardness of the method.

The article is organized as follows: In Section 2, the MSE method is discussed. In Section 3 the MSE method is applied to find exact traveling wave solutions of the nonlinear evolution equations pointed out above; in Section 4, comparison between the MSE method and the Exp-function method for nKG equation is discussed, and in Section 5 conclusions are given.

2. Description of MSE method

Suppose we have a nonlinear evolution equation in the form,

$$\wp(u, u_t, u_x, u_{xx}, u_{tt}, \ldots) = 0, \quad x \in R \text{ and } t \geqslant 0, \tag{2.1}$$

where $\wp$ is a polynomial of $u(x, t)$ and its partial derivatives wherein the highest order derivatives and nonlinear terms are involved. The focal steps of the MSE method [28–30] are as follows:

Step 1: The traveling wave transformation [21],

$$u(x,t) = u(\xi), \quad \xi = k(x \pm \omega t), \tag{2.2}$$

where ω is the speed of traveling wave, k is the wave number.

The traveling wave transformation Eq. (2.2) permits us to transform Eq. (2.1) into the following ordinary differential equation (ODE):

$$P(u, u', u'', \ldots) = 0, \tag{2.3}$$

where P is a polynomial in $u(\xi)$ and its derivatives, forasmuch $u'(\xi) = \frac{du}{d\xi}$.

Step 2: Assume that the formal solution of Eq. (2.1) can be expressed in the form:

$$u(\xi) = A_0 + \sum_{i=1}^{N} A_i \left(\frac{\Phi'(\xi)}{\Phi(\xi)}\right)^i, \tag{2.4}$$

where N is a positive integer, $A_N \neq 0$, and A_i $(i = 1, 2, 3, \ldots, N)$ are arbitrary constants to be determined, and $\Phi(\xi)$ is an unknown function to be determined afterward, such that $\Phi'(\xi) \neq 0$.

In the tanh-function method [16,20], the (G'/G)-expansion method [6,4,5,21], the Exp-function method [12,7,19,18], and so forth, the solution is offered in terms of some predefined functions, but in the MSE method, Φ is not predefined or not a solution of any predefined differential equation. This is the good point of the MSE method.

Step 3: The positive integer N that occurs in Eq. (2.4) can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (2.1) or Eq. (2.3). Moreover, the degree of $u(\xi)$ is defined as $D(u(\xi)) = N$ which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = N + q, \quad D\left(u^p \left(\frac{d^q u}{d\xi^q}\right)^s\right) = Np + s(N+q). \tag{2.5}$$

Therefore, it is easy to find the value of N in Eq. (2.4), using Eq. (2.5).

Step 4: Substitute Eq. (2.4) into (2.3), and calculate all the necessary derivatives $u', u'', \ldots$ of the unknown function $u(\xi)$ and then account the function $\Phi(\xi)$. As a result of this substitution, a polynomial of Φ^{-j}, $(j = 0, 1, 2, \ldots)$ with the derivatives of $\Phi(\xi)$ will be found. In this polynomial equate all the coefficients of Φ^{-j} to zero, where $j = 0, 1, 2, \ldots$. This procedure yields a system of algebraic and ordinary differential equations. The values of A_N's can be found by solving the algebraic equations, and to determine $\Phi(\xi)$, ODEs are needed to solve. Substituting the values of A_N and $\Phi(\xi)$ into Eq. (2.4) completes the determination of the solution of Eq. (2.1).

3. Applications

3.1. The nonlinear Klein–Gordon equation

In this sub-section, the MSE method is used to find the exact solutions of the nKG equation [23],

$$u_{tt} - u_{xx} + \alpha u + \beta u^3 = 0, \tag{3.1}$$

where α and β are nonzero parameters.

It arises in many physical problems including nonlinear dispersion [33,34] and nonlinear meson theory [35,36].

The traveling wave transformation,

$$u(x,t) = u(\xi), \qquad \xi = k(x - \omega t), \tag{3.2}$$

transforms the Eq. (3.1) to the following ODE:

$$k^2(\omega^2 - 1)u'' + \alpha u + \beta u^3 = 0 \tag{3.3}$$

Balancing the highest order derivative u'' and nonlinear term of the highest order u^3, yields $N + 2 = 3N$ which gives $N = 1$.

Therefore, solution Eq. (2.4) becomes,

$$u(\xi) = A_0 + A_1\left(\frac{\Phi'}{\Phi}\right), \tag{3.4}$$

where A_0 and A_1 are constants such that $A_1 \neq 0$, and $\Phi(\xi)$ is an unidentified function to be determined. It is easy to calculate that,

$$u' = A_1\left[\frac{\Phi''}{\Phi} - \left(\frac{\Phi'}{\Phi}\right)^2\right]. \tag{3.5}$$

$$u'' = A_1\frac{\Phi'''}{\Phi} - 3A_1\frac{\Phi''\Phi'}{\Phi^2} + 2A_1\left(\frac{\Phi'}{\Phi}\right)^3. \tag{3.6}$$

$$u^3 = A_1^3\left(\frac{\Phi'}{\Phi}\right)^3 + 3A_1^2A_0\left(\frac{\Phi'}{\Phi}\right)^2 + 3A_1A_0^2\left(\frac{\Phi'}{\Phi}\right) + A_0^3. \tag{3.7}$$