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Superconductivity in the graphene monolayer calculated using the Kubo formulalism

L.S. Lima

Departamento de Física e Matemática, Centro Federal de Educação Tecnológica de Minas Gerais, Belo Horizonte, MG 30510-000, Brazil

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ABSTRACT

We have employed the massless Dirac's fermions formalism together with the Kubo's linear response theory to study the transport by electrons in the graphene monolayer. We have calculated the electric conductivity and verified the behavior of the *AC* and *DC* electric conductivities of the system that is known to be a relativistic electron plasma. Our results show a superconductor behavior to the electron transport and consequently the spin transport for all values of $T > 0$ and a behavior of the *AC* conductivity tending to infinity in the limit $\omega \to 0$. In $T = 0$ our results show an insulator behavior with a transition from a superconductor state at $T > 0$ to an insulator state at $T = 0$.

1. Introduction

Graphene is an allotropic form of the carbon that is a lot researched actually. Its semiconductor properties with low-lying excitations obey the massless Dirac's equation [1]. The interplay between the antiferromagnetic state and the Kekule valence bond solid ordering in the zero energy levels of neutral monolayer and bilayer graphene have been studied in Ref. [2]. The understanding of the dynamics of many interacting particles is a formidable task in physics. For electronic transport in matter, strong interactions can lead to a breakdown of Fermi liquid paradigm of coherent quasi-particles scattering of impurities. In such situations, provided that certain conditions are filled, the complex microscopic dynamics can be coarse-grained to a hydrodynamics description of the momentum, energy and charge transport on long length and time scales [3,4].

The spin transport properties by electrons in graphene has been studied in the literature using the Boltzmann's equation formalism [5,6]. Is well known that a spin current can be converted in a charge current at room temperature [7–9]. Graphene is an interesting material for spintronics showing long spin relaxation lengths even at room temperature. For future spintronic devices is important to understand the behavior of the spins and the limitations for the spin transport in structures where the dimensions are smaller than the spin relaxation length [10]. The electron spin lifetime in carbon materials is expected to be very long because of the very large natural abundance of the isotope 12C without nuclear spin and a small size of spin orbit coupling. This even led to propose graphene as an optimal material to store quantum information in the spins of confined electrons. Moreover the most of the experiments show that the spin lifetimes are in the range of nanoseconds, shorter than expected from these considerations, which lies at the heart of the design of devices where graphene is used as a passive component to carry electron currents spin polarized. The electronic properties of transition metal atoms adsorbed on a graphene sheet have been analyzed in the framework of the quantum theory of atoms and molecules [11].

The Kubo formalism has been many employed in the literature to study the spin transport by ions of the lattice in spin systems described by the Heisenberg model and XY model. For example, Sentef et al. [12] has analyzed the spin transport in the easy-axis Heisenberg antiferromagnetic model in two and three dimensions. Damle and Sachdev [13] treated the two-dimensional case using the non-linear sigma model in the gapped phase. Pires and Lima [14–17] treated the one and two-dimensional easy plane Heisenberg antiferromagnetic model. Lima and Pires [18] studied the spin transport in the two-dimensional anisotropic XY model using the SU(3) Schwinger boson theory in the absence of impurities, Lima [19] studied the case of the Heisenberg antiferromagnetic model in one and two dimensions with Dzyaloshinskii–Moriya interaction. Zewei Chen et al. [20] analyzed the effect of spatial and spin anisotropy on spin conductivity for the $S = 1/2$ Heisenberg model on a square lattice and more recently, Kubo et al, [21] studied the spin transport in two-dimensional non-collinear antiferromagnets at $T = 0$ using spin wave theory, Lima has studied the spin transport in the site diluted two-dimensional anisotropic Heisenberg model in the easy plane using the self consistent harmonic approximation and the Schwinger boson theory [22–27].

E-mail address: lslima@fisica.ufmg.br

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The electron transport properties of zigzag graphene nanoribbons with upright standing carbon chains were investigated by using first-principles calculations. The calculated results show a significant odd-even dependence [28]. From an experimental point of view, recently, there is an intense research about spin transport by electrons where has been investigated the quantum Hall effect for spins and the magnon spintronics [29–35]. In studies of these effects often only the sign differences between related quantities like magnetic fields and generated spin and charge currents are determined. The spin injection and transport in single layer graphene can be investigated using nonlocal magnetoresistance (MR) measurements [36–40].

The aim of this paper is to study the spin transport by electrons in graphene monolayer using the Dirac's fermions formalism. The graphene consists in a fermion system with a relativistic Dirac spectrum where the energy vanishes linearly at isolated points in the first Brillouin zone. Dirac's fermions are provided by numerous new experimental realizations. These include d-wave superconductors and topological insulators [41].

This work is divided in the following way. In Section 2, we discuss the Dirac's fermion model, in Section 3 we discuss the Kubo's formalism for the electron transport and we present the analytical results, in Section 4 we make the diagrammatic expansion for the Green's function and in the last section, Section 5, we present our conclusions and final remarks.

2. The Dirac fermion model

The model of free-fermionic particles relativistic of the graphene in $D = 1+1$ dimension is described by the following Hamiltonian density [42]

$$\mathcal{H} = v_F J \int d^2x \psi^{\dagger}_{\alpha}(x) i\sigma_3^{\alpha\beta} \partial_x \psi_{\beta}(x), \tag{1}$$

where $\psi_{\alpha}(x)$ with $\alpha, \beta = 1, 2$ denote a two component Fermi field in $D = 1+1$ and $D = 2+1$ space dimensions and v_F is the Fermi's velocity. We have considered unities where $v_F = 1$ and $\hbar = 1$. The interaction term has the form up to the irrelevant additive constant

$$\mathcal{H}_{int} = -2\gamma J \int d^2x (\bar{\psi}(x)\psi(x))^2 \tag{2}$$

that is the interaction term of the (1+1)-dimensional Gross–Neveu-model. The expression $\bar{\psi}\psi$ is the continuum limit of

$$\frac{1}{2a_0}(n(2s+1) - n(2s)) \approx -\left(\psi_1^{\dagger}(x)\psi_1(x) - \psi_2^{\dagger}(x)\psi_2(x)\right) \equiv \bar{\psi}\psi, \tag{3}$$

a_0 is the lattice spacing and σ_3 is the diagonal Pauli matrix.

$$\psi_1(x) = \frac{1}{\sqrt{2}}(-R(x) + L(x))$$
$$\psi_2(x) = \frac{1}{\sqrt{2}}(R(x) + L(x)) \tag{4}$$

where L and R are the Fermi fields moving towards the right and left respectively with speed $v_F = 1$.

The single-particle spectrum has the relativistic form

$$\omega_k \simeq v_F k, \tag{5}$$

where in the massless (gapless) limit, the spectrum is linear, therefore time and space scale in the same way $T \sim L$ as required by relativistic invariance [42]. There are local interactions such as $(\bar{\psi}\gamma_{\mu}\psi)^2$ and $(\bar{\psi}\psi)$, where $\bar{\psi} = \psi^{\dagger}\gamma^0$. The action of a free Dirac's field is given by

$$S = \int d^2x \bar{\psi}_{\alpha}(x) i\gamma^{\mu}_{\alpha\beta}\partial_{\mu}\psi_{\beta}(x) \tag{6}$$

where γ are the 4×4 Dirac's matrixes.

$$\gamma^0 = -i\begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix} \qquad \gamma = -i\begin{pmatrix} 0 & -\sigma \\ -\sigma & 0 \end{pmatrix}. \tag{7}$$

$\mathbf{1}$ is the unit 2×2 matrix, and σ are the components of Pauli's matrices where the Dirac γ-matrices, γ_0, γ_1 and γ_5 satisfy

$$\{\gamma_{\mu}, \gamma_{\nu}\} = 2g_{\mu\nu}, \quad \gamma_5 = i\gamma_0\gamma_1, \tag{8}$$

where $g_{\mu\nu}$ is the metric tensor. In the theory of free massless Dirac's fermions there is a fixed point of the renormalization group [42].

3. The Kubo formalism of transport

We use the low energy approach Dirac's fermion [5,42] to determine the regular part of the electron conductivity (*AC* conductivity) or continuum conductivity . An electron current appears if there is an electric field, which is given by the Ohm' Law $\mathcal{J} = \sigma\vec{E}$. In a similar way a spin current appears as a response to a magnetic field $\mathcal{J}_S = \sigma\nabla\vec{B}$, through the system, where it plays the role of a chemical potential for spins. If we connect a low dimensional magnet with two bulk ferromagnetic, they can act as reservoirs for spins [33,34]. Then, one has a flow of spin current if there is a difference, $\Delta\vec{B}$, between the magnetic fields at the two ends of the sample.

In the Kubo formalism [12,14,43,44] the electric conductivity or the spin conductivity is given by:

$$\sigma(\omega) = \lim_{\vec{k}\to 0} \frac{\langle\mathcal{K}\rangle + \Lambda(\vec{q}, \omega)}{i(\omega + i0^+)}, \tag{9}$$

where $\langle\mathcal{K}\rangle$ is the kinetic energy and $\Lambda(\vec{q}, \omega)$ is the current-current correlation function defined by

$$\Lambda(\vec{k}, \omega) = \frac{i}{\hbar N}\int_0^{\infty} dt e^{i\omega t}\langle[\mathcal{J}(\vec{k}, t), \mathcal{J}(-\vec{k}, 0)]\rangle. \tag{10}$$

$\Lambda(\vec{k}, \omega + i0^+)$ is analytic in the upper half of the complex plane and extrapolation along the imaginary axis can be reliably done.

The current operator for graphene is given by [5,42]

$$\mathcal{J} = \bar{\psi}\gamma_{\mu}\psi. \tag{11}$$

The real part of $\sigma(\omega)$, $\sigma'(\omega)$, can be written in a standard form as [44]

$$\sigma'(\omega) = \sigma_0(\omega) + \sigma^{reg}(\omega), \tag{12}$$

where $\sigma_0(\omega)$ is the *DC* contribution given by $\sigma_0(\omega) = D_S\delta(\omega)$, here D_S is the Drude's weight

$$D_S = \pi[\langle\mathcal{K}\rangle + \Lambda'(\vec{k} = 0, \omega \to \vec{0})]. \tag{13}$$

$\sigma^{reg}(\omega)$, the regular part of $\Re\sigma(\omega)$, is given by [44]

$$\sigma^{reg}(\omega) = \frac{\Lambda''(\vec{k} = 0, \omega)}{\omega}. \tag{14}$$

It represents the continuum contribution to the conductivity. In the Eqs. (13) and (14), Λ' and Λ'' stands for the real and imaginary part of Λ.

Using the Matsubara's method, we obtain the Green's function for the model Eq. (1) as

$$G_j(\omega) = \sum_{\alpha}\int_0^{\pi}\frac{d^2k}{(2\pi)^2} v_F^2 G_{\alpha}^0(\omega_1)\tilde{G}_{\alpha}^0(\omega + \omega_1) \tag{15}$$

with

$$G_{\alpha}^0(\omega) = \frac{1}{i\omega_n - \omega_k}, \qquad \tilde{G}_{\alpha}^0(\omega) = \frac{-1}{i\omega_n + \omega_k}. \tag{16}$$