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Negative differential resistance in Josephson junctions coupled to a cavity

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ABSTRACT

Regions with negative differential resistance can arise in the IV curve of Josephson junctions and this phenomenon plays an essential role for applications, in particular for THz radiation emission. For the measurement of high frequency radiation from Josephson junctions, a cavity – either internal or external – is often used. A cavity may also induce a negative differential resistance region at the lower side of the resonance frequency. We investigate the dynamics of Josephson junctions with a negative differential resistance in the quasi particle tunnel current, i.e. in the McCumber curve. We find that very complicated and unexpected interactions take place. This may be useful for the interpretation of experimental measurements of THz radiation from intrinsic Josephson junctions.

1. Introduction

Stacks of BSCCO consisting of coupled Josephson Junctions (JJ) can give rise to interesting emission in the THz region [1–4]. The technological applications of THz electromagnetic radiation span in the field of security, nondestructive sensing, biological sensors, fast communications, and analogical processing [5]. Several technologies for THz management have been proposed. However, such frequency range is difficult to access, because it belongs to the so-called "terahertz gap".

The interaction between JJ and the cavity is therefore an essential step for applications, inasmuch as the power emitted by the JJ is coupled to an external load that, at a simple level, can be modeled by an *RLC* circuit [6]. For instance, a characterization of the radiation power emitted by an antenna has been proposed in Ref. [7], where it has been shown that the antenna is well described by an *RLC* circuit.

JJ emission is associated with Negative Differential Resistance (NDR) in the *RLC* circuit together with the nonlinear Josephson junction [8,9]. In this work we will examine the interplay between the coupling circuit *RLC* and the JJ with NDR in the resistive part of the IV curve. Finally, we mention that arrays of JJ [10,11] are important for applications, as a single junction does not provide enough power for many practical purposes. The NDR considered in this work is a phenomenological approach to the more complicated dynamics arising in JJ arrays interaction [12,13].

2. Model

The resistively shunted Josephson junction (RSJ) equivalent circuit coupled to a cavity is schematically depicted in Fig. 1a [6]. The phase difference across the Josephson barrier is denoted $\phi = \phi(t)$ and the voltage across the junction is given by $V(t) = \hbar/(2e) d\phi/dt$, where $\hbar$ is Planck's constant divided by 2π and e is the electron charge. The Josephson critical current is I_0, C_J is the junction capacitance and R_J denotes the quasiparticle tunneling resistance. The bias current is I. For the external linear cavity coupled to the Josephson junction circuit, the charge on the cavity capacitor is $\tilde{q} = \tilde{q}(t)$ and the cavity capacitance is C. In the cavity circuit we have inserted a resistor with resistance R and inductor with inductance L. For a Josephson junction with a linear cavity and linear shunt resistance (McCumber resistance) we get [7,14,15]:

$$\frac{C_J\hbar}{2e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{R_J 2e}\frac{d\phi}{dt} + I_0 \sin\phi + \frac{d\tilde{q}}{dt} = I \quad (1)$$

$$\frac{d^2\tilde{q}}{dt^2} + \frac{R}{L}\frac{d\tilde{q}}{dt} + \frac{1}{LC}\tilde{q} - \frac{\hbar}{2eL}\frac{d\phi}{dt} = 0. \quad (2)$$

Introducing the Josephson plasma frequency $\omega_0 = \sqrt{2eI_0/C_J\hbar}$, Eq. (1) can be cast in normalized units $\tau = \omega_0 t$ and $q = \omega_0\tilde{q}/I_0$:

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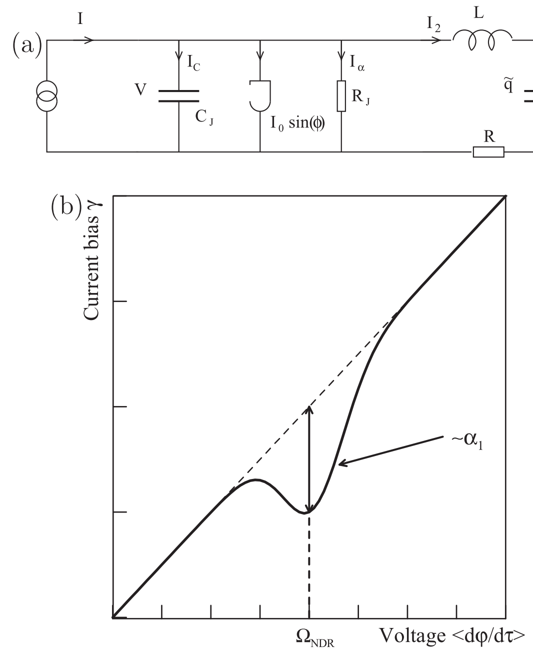

Fig. 1. (a) Equivalent circuit of a JJ coupled to a cavity. (b) The IV curve of the nonlinear conductance, Eq. (5). The dashed line is a linear conductance ($\alpha_1 = 0$).

$$\frac{d^2\phi}{d\tau^2} + \alpha\frac{d\phi}{d\tau} + \sin\phi + \frac{dq}{d\tau} = \gamma, \tag{3}$$

$$\frac{d^2q}{d\tau^2} + \frac{\Omega_r}{Q}\frac{dq}{d\tau} + \Omega_r^2 q - \beta_L\frac{d\phi}{d\tau} = 0, \tag{4}$$

where

$$\frac{Q}{\Omega_r} = \frac{L}{R}\sqrt{\frac{2eI_0}{C_J\hbar}}, \ \beta_L = \frac{2eLI_0}{\hbar}, \ \gamma = \frac{I}{I_0}, \ \Omega_r = \frac{1}{\omega_0\sqrt{LC}}.$$

This model can be modified to account for a nonlinear conductance observed in experiments and in simulations of stacks of Josephson junctions [12,13]. A possibility is to introduce the nonlinear dissipation coefficient [16] in normalized units as follows

$$\alpha(d\phi/d\tau) = \alpha_0\left[1 - \alpha_1 \exp\left(-\frac{1}{\Delta}\left(\frac{d\phi}{d\tau} - \Omega_{NDR}\right)^2\right)\right], \tag{5}$$

that models the effect of the complicated interaction in the BSCCO stack. Here $\alpha_0 = \frac{1}{R_J}\sqrt{\frac{\hbar}{2eI_0C_J}}$, is the normalized linear quasi particle conductance and α_1 is a measure of the height of the Gaussian, see Fig. 1. The parameter Ω_{NDR} determines the normalized voltage where α takes its minimum value, while the parameter Δ is connected to the width of the NDR region. Examples of a nonlinear McCumber curve could be at the energy gap in intrinsic Josephson junctions and at the gap difference in junctions made of different materials.

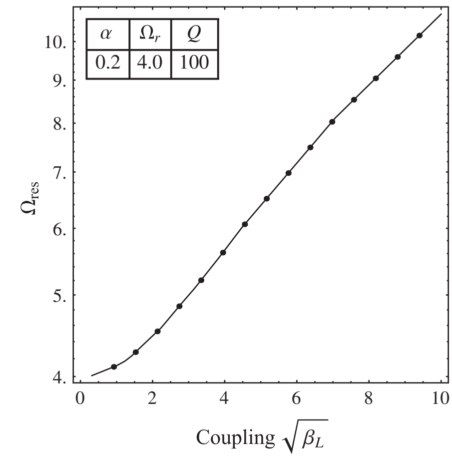

Fig. 3. The resonant frequency Ω_{res} as a function of the square root of the coupling $\sqrt{\beta_L}$.

The modified conductance changes the Josephson voltage relation and Eq. (4) becomes

$$\frac{d^2\phi}{d\tau^2} + \alpha\left(\frac{d\phi}{d\tau}\right)\frac{d\phi}{d\tau} + \sin(\phi) = -\frac{d\tilde{q}}{d\tau} + \gamma_B. \tag{6}$$

That reduces to the linear case for $\alpha_1 = 0$.

3. Results

To understand the combined effect of a JJ and an *RLC* we start simulating the IV curve and the current oscillations in the *RLC* circuit. To estimate the power available in the cavity we introduce the quantity.

$$A = \max_\tau \frac{dq}{d\tau} - \min_\tau \frac{dq}{d\tau} \tag{7}$$

We start with the NDR $\alpha_1 = 0.5$. An example is shown in Fig. 2. From the figure it is evident that the resonance of the *RLC* circuit is shifted to higher values, $V \simeq 5$ with respect to the resonance

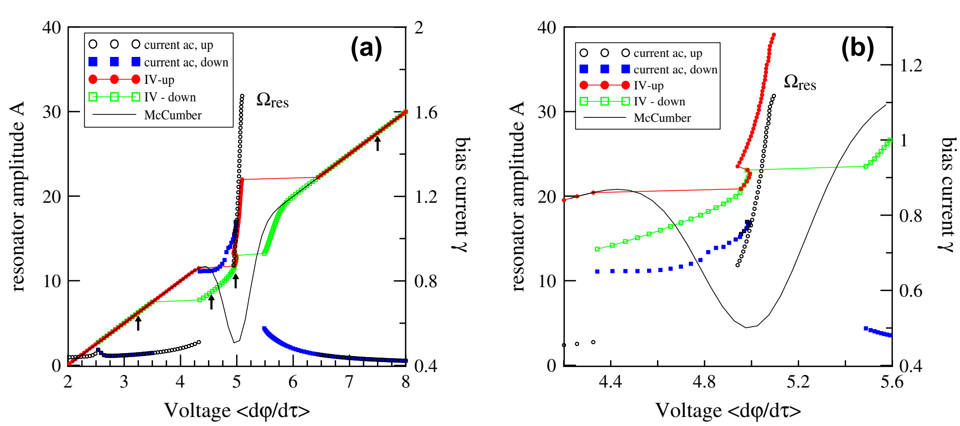

Fig. 2. The IV curve and the current in the *RLC* resonator. The bias current is swept from low and high values (a) and an enlargement of the central area where the backbending analogous to the phenomenon observed in stacks of BSCCO [8]. The parameters of the simulations are: $\alpha_0 = 0.2$, $\alpha_1 = 0.5$, $\Omega_{NDR} = 5$, $\Delta = 0.1$, $\beta_L = 10$, $Q = 100$, $\Omega_r = 4$.

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