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Magnetic field expulsion from an infinite cylindrical superconductor

Miguel C.N. Fiolhais [a,*], Hanno Essén [b]

[a] LIP, Department of Physics, University of Coimbra, 3004-516 Coimbra, Portugal
[b] Department of Mechanics, KTH, 10044 Stockholm, Sweden

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ABSTRACT

The solutions of the London equations for the magnetic field expulsion from superconductors are presented in this paper for the cylindrical symmetry. The result is analyzed in detail and represented numerically for the case of a uniform external magnetic field in the transverse plane. In particular, several contour plots of the magnetic energy density are depicted for the regions inside and around the superconducting area for a wide range of penetration lengths, showing how the expulsion and penetration of the magnetic field evolve with the ratio between the penetration length and the cylinder radius.

1. Introduction

The first phenomenological description of the magnetic field expulsion from superconductors appeared soon after the discovery of the Meissner-Ochsenfeld effect [1], by the London brothers in 1935 [2]. Ever since, several discussions have been presented along the years on the nature of the magnetic field expulsion configuration in the scientific literature [3–17]. These studies comprise both static and time-dependent analyzes, several different geometries, and even the case of non-homogenous superconductors. However, while the spherical case is studied in [17–19], and the cylinder case in a parallel magnetic field is presented in [19], to the best of our knowledge the detailed solution of a superconducting cylinder in a transversal magnetic field cannot be easily found in the academic literature. This very same case has been previously addressed by Zhilichev [7] by proposing a macroscopic shell model, but without providing an explicit solution for the London condition. To fill this gap, we derive the solution of the London equations in this paper, for the magnetic field expulsion from an infinite homogeneous superconducting cylinder in a constant external magnetic field in the transverse plane. The dependence of the strength and configuration of the expelled magnetic field with the London penetration depth is examined in detail.

The London equations for the magnetic field flux expulsion out of a superconductor can be expressed, in terms of the magnetic vector potential $\mathbf{A}$, in a single equation:

$$\mathbf{j} = -\frac{1}{\lambda^2}\mathbf{A}, \tag{1}$$

where $\mathbf{j}$ represents the electric current and λ the London penetration depth. After applying the Maxwell's equation for the static case, and by taking the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$), one obtains:

$$\nabla^2\mathbf{A} - \frac{1}{\lambda^2}\mathbf{A} = 0, \tag{2}$$

or, in terms of the magnetic field,

$$\nabla^2\mathbf{B} - \frac{1}{\lambda^2}\mathbf{B} = 0. \tag{3}$$

This equation is perhaps the most common form of the London equation for the magnetic field expulsion inside the superconducting region, and, in the cartesian coordinate system, corresponds to the well-known Helmholtz's equation for a complex wavenumber, for each of the vector components. The magnetic field is, therefore, suppressed in the interior region through negative exponential dependencies, or, in the cylindrical and spherical symmetries, through modified Bessel functions of the first kind [20].

2. Cylindrical symmetry

Consider a cylindrical superconductor with radius R in an external constant perpendicular magnetic field, in such a way the external magnetic field points in the y-direction and the cylinder axis coincides with the z-axis. In order to obtain the final static vector potential configuration resulting from the magnetic flux expulsion, and assuming the Coulomb gauge, one must solve the Maxwell equation for the outer region:

* Corresponding author. Tel.: +351 964590914.
E-mail addresses: miguel.fiolhais@cern.ch (M.C.N. Fiolhais), hanno@mech.kth.se (H. Essén).

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$$\nabla \times \mathbf{B} = \nabla \times (\nabla \times \mathbf{A}) = \nabla^2 \mathbf{A} = 0, \tag{4}$$

and the London Eq. (2) for the inner region. Due to the symmetry of the system, the magnetic field has no component in the z-axis direction,

$$\mathbf{B} = \frac{1}{\rho}\frac{\partial A_z}{\partial \phi}\hat{\rho} - \frac{\partial A_z}{\partial \rho}\hat{\phi}, \tag{5}$$

and, therefore, the axial component of the vector potential A_z is the only relevant component needed to compute the magnetic field. For the outer region, this component can be determined from Eq. (4):

$$\left(\frac{1}{\rho^2}\frac{\partial^2 A_z}{\partial \phi^2} + \frac{\partial^2 A_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A_z}{\partial \rho}\right)\hat{z} = 0, \tag{6}$$

which is simply the Laplace equation for A_z. The most general solution in the outer region is, therefore,

$$A_z = \sum_{n=0}^{+\infty}[A_n \cos(n\phi) + B_n \sin(n\phi)] \times [C_n \rho^n + D_n \rho^{-n}]. \tag{7}$$

Assuming the magnetic field takes the form of the external field in the limit $\rho \to +\infty$,

$$\mathbf{B}_0 = B_0 \hat{\mathbf{y}} = B_0\left(\hat{\phi}\cos\phi + \hat{\rho}\sin\phi\right), \tag{8}$$

and in order to respect this boundary condition, the vector potential becomes,

$$A_z = \text{const.} + A_1(C_1\rho + D_1\rho^{-1})\cos\phi, \tag{9}$$

where $A_1 C_1 = -B_0$. The remaining parameters must be determined by the boundary conditions at the surface of the superconductor.

The general solution of the London equation in the interior region, for the axial component of the vector potential, is somewhat more complicated:

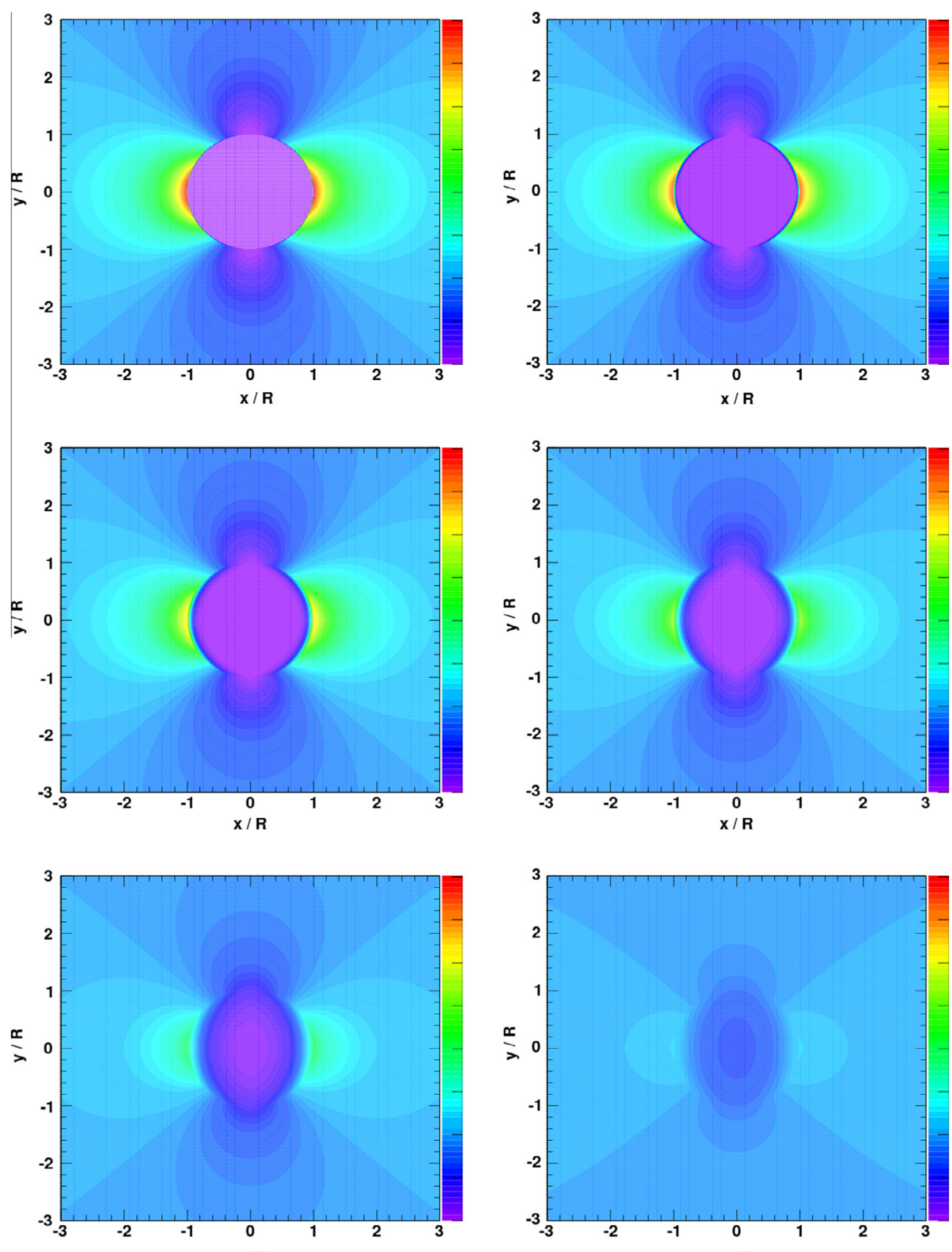

Fig. 1. Contour plots of the magnetic energy density for $\lambda = 0$ (top left), $\lambda = 0.03R$ (top right), $\lambda = 0.1R$ (center left), $\lambda = 0.2R$ (center right), $\lambda = 0.4R$ (bottom left), and $\lambda = R$ (bottom right).