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ORIGINAL ARTICLE

Hydromagnetic effect on inclined peristaltic flow of a couple stress fluid

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Abstract In this paper, we have investigated the effect of channel inclination on the peristaltic transport of a couple stress fluid in the presence of externally applied magnetic field. The slip velocity at the channel wall has been taken into account. Under the long wave length and low-Reynolds number assumptions, the analytical solutions for axial velocity, stream function, pressure gradient and pressure rise are obtained. The computed results are presented graphically by taking valid numerical data for non-dimensional physical parameters available in the existing scientific literatures. The results revealed that the trapping fluid can be eliminated and the central line axial velocity can be reduced with a considerable extent by the application of magnetic field. The flow phenomena for the pumping characteristics, trapping and reflux are furthermore investigated. The study shows that the slip parameter and Froude number play an important role in controlling axial pressure gradient.

1. Introduction

It is well known that mixing and transporting of physiological fluids is referred as peristalsis, which is generated due to progressive waves of area contraction and expansion along the length of a distensible tube containing fluid. The mechanism behind this phenomenon is mainly neuromuscular property of any tubular smooth muscle structure. This type of muscular tube wall has a motion in wave frame with a fixed speed and wave length. This mechanism is found in urine transport from kidney to the bladder, the movement of chime into the gastrointestinal tract, fluids in the lymphatic vessels, bile from the gallbladder into the duodenum, the movement of spermatozoa in the ducts efferent of the male reproductive tract, the movement of the ovum in the fallopian tube and the circulation of blood in small blood vessels. This mechanism also finds many applications in bio-medical engineering to design roller and finger pumps, some bio-mechanical instruments, e.g., heart–lung machine, blood pump machine and dialysis machine.

Latham [1] first initiated the concept of peristaltic mechanism. Later on, this mechanism has become an important topic of research owing to the above mentioned applications in biomechanical engineering and biomedical technology. Several investigators [2–5] have studied the peristaltic transport of fluids in tubes for better and clear understanding of peristaltic

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mechanism. Weinberg et al. [6] conducted an experimental study of peristaltic pumping, whereas Yin and Fung [7] made a comparison between theoretical and experimental studies in peristaltic transport. Misra and Pandey [8] gave a mathematical model for peristaltic transport of blood in small vessels. Further, Riaz et al. [9] studied the peristaltic transport of a Carreau fluid in a compliant rectangular duct by utilizing the assumptions of long wave length and low Reynolds number. Mekheimer et al. [10] put forwarded peristaltic motion of a particle–fluid suspension in a planar channel. They used perturbation technique to solve the problem by assuming wave number to be small. Many authors [11–14] have studied on the peristaltic transport in an asymmetric channel by considering different kind of fluid models, wherein they found distinguishable effect of phase difference of wall motion on velocity and other flow characteristics. Mekheimer [15] presented a theoretical study on peristaltic transport of a Newtonian fluid through a uniform and non-uniform annulus. Nadeem et al. [16] proposed a series solution of unsteady peristaltic flow of an incompressible Carreau fluid in eccentric cylinders, whereas Akbar et al. [17] carried out a numerical simulation of peristaltic flow of a Carreau nanofluid in an asymmetric channel. Elad [18] have developed a mathematical model for wall-induced peristaltic fluid flow in a channel with wave trains having phase difference on the upper and lower walls. The studies on the different non-Newtonian fluid models were carried out by several investigators [19–22]. Misra et al. [23] studied the peristaltic transport of a physiological fluid in an asymmetric porous channel in the presence of an external magnetic field. However, Shit and Roy [24] investigated the effect of induced magnetic field on peristaltic flow a micropolar fluid in an asymmetric channel.

The couple-stress fluid may be considered as a special class of a non-Newtonian fluid, which takes into account the effect of particle size. To characterize the couple stress fluid, Stokes [25] gave a concept of constitutive relationship between the stress and strain rate in micro-continuum theory of fluids which allows for polar effects such as the presence of couple stresses, body couples and a non-symmetric stress tensor. The constitutive equations in these fluid models are very complex because of the involvement of various material constants leading to a boundary value problem so that the order of the differential equations is higher than the Navier–Stokes equations. Valanis and Sun [26] and Pal et al. [27] studied on the couple stress fluid having applications in blood flow through cardiovascular system. Srivastava [28] have examined the peristaltic transport of a couple stress fluid under a zero Reynolds number and long wave length approximations. Rao and Rao [29] put forwarded the peristaltic flow of a couple stress fluid through a porous medium in a channel at low Reynolds number. Mekheimer [30] and Nadeem and Akram [31] have investigated the peristaltic transport of a couple stress fluid in different geometrical situations. Sobh [32] analytically studied the interaction of Couple stresses and slip flow on peristaltic transport in uniform and non-uniform channels. Recently Pandey and Choube [33] examined the effect of magnetic field on peristaltic transport of couple stress fluids through a porous medium. Eldabe [34] studied on the MHD peristaltic flow of a couple stress fluids with heat and mass transfer phenomena. Moreover, Sankad and Radhakrisnamacharya [35] contributed toward the study of effect of magnetic field on the peristaltic transport of couple stress fluid in a channel with different wall properties. Kothandapani [36] have considered the non-linear peristaltic transport of a Newtonian fluid in an inclined asymmetric channel through porous medium. However, to the best of author's knowledge, no one has considered the slip effect on peristaltic transport of couple stress fluid in an inclined asymmetric channel along with the externally applied magnetic field.

In this paper, we have analyzed the effects of slip velocity as well as magnetic field on peristaltic flow of couple stress fluid in an inclined asymmetric channel. The analytical solutions to the axial velocity, stream function and pressure rise per wave length are obtained using the assumptions of long wave length and low Reynolds number. Finally, the numerical solutions have been computed using MATHEMATICA software and presented them graphically. Therefore, our theoretical investigation bears the potential to useful in the field of bio-fluid dynamics.

2. Problem formulation

Let us consider the flow of an incompressible, viscous and electrically conducting couple-stress fluid flowing through an inclined asymmetric channel of uniform thickness under the action of an external magnetic field. Let $Y' = h'_1$ and $Y' = h'_2$ represent respectively the upper wall and lower wall of the asymmetric channel (cf. [11–13,24]). The medium is considered to be induced by a sinusoidal wave train propagating with a wave speed c along the length of the channel wall (cf. Fig. 1), such that

$$h'_1(X', t') = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(X' - ct')\right], \tag{1}$$

$$h'_2(X', t') = -d_2 - a_2 \cos\left[\frac{2\pi}{\lambda}(X' - ct') + \phi\right], \tag{2}$$

where d_1 and d_2 are the mean height of the upper and lower wall of the channel from the central line, a_1 and $a_2$are the amplitudes of waves of the channel walls, λ the wave length, $\phi(0 \leqslant \phi \leqslant \pi)$ the phase difference between the wave trains of both the walls, X' and Y' are the rectangular co-ordinates with X' measures the axis of the channel and Y' the traverse axis perpendicular to X'.

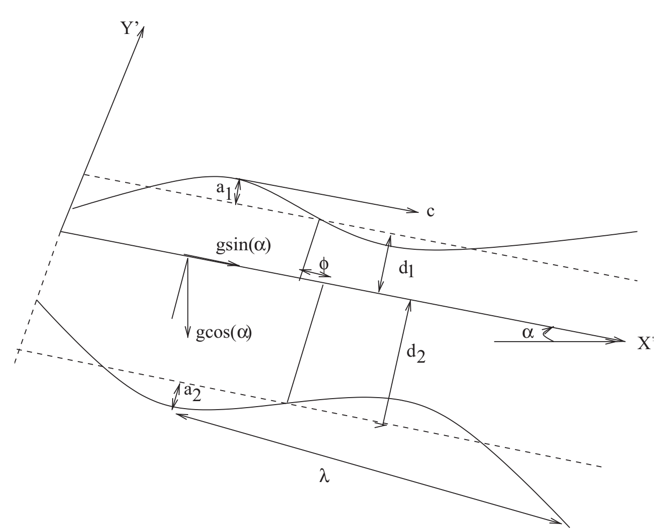

Fig. 1 A physical sketch of the problem.