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Intelligent control for nonlinear inverted pendulum based on interval type-2 fuzzy PD controller

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Abstract The interval type-2 fuzzy logic controller (IT2-FLC) is able to model and minimize the numerical and linguistic uncertainties associated with the inputs and outputs of a fuzzy logic system (FLS). This paper proposes an interval type-2 fuzzy PD (IT2F-PD) controller for nonlinear inverted pendulum. The proposed controller uses the Mamdani interval type-2 fuzzy rule based, interval type-2 fuzzy sets (IT2-FSs) with triangular membership function, and the Wu–Mendel uncertainty bound method to approximate the type-reduced set. The proposed controller is able to minimize the effect of the structure uncertainties and the external disturbances for the inverted pendulum. The results of the proposed controller are compared with the type-1 fuzzy PD (T1F-PD) controller in order to investigate the effectiveness and the robustness of the proposed controller. The simulation results show that the performance of the proposed controller is significantly improved compared with the T1F-PD controller. Also, the results show good performance over a wide range of the structure uncertainties and the effect of the external disturbances.

1. Introduction

The classical linear proportional-derivative (PD) controller and the proportional-integral-derivative (PID) controller cannot provide good enough performance in controlling highly complex, nonlinear, and uncertain processes [1,2]. Fuzzy control [3,4] has become, in the recent past, an alternative to conventional control algorithms to deal with complex processes and combine the advantages of classical controllers and human operator experience. The main advantage of a FLC is that it can be applied to systems that are nonlinear where their mathematical models are difficult to obtain. Another advantage is that the controller can be designed to apply heuristic rules that reflect the experiences of the human experts [5]. A suitable choice of control variables is important in fuzzy control design. Typically, the inputs to the fuzzy controllers are the error and the change of error. This choice is physically related to classical PID controllers. Usually, a fuzzy controller is either a PD- or a

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PI-type depending on the output of fuzzy control rules; if the output is the control signal it is said to be a PD-type fuzzy controller and if the output is the change of control signal it is said to be a PI-type fuzzy controller [6].

Despite the significant improvement in these fuzzy controllers over their conventional counterparts, it should be noted that they are usually not effective if the system to be controlled has structure uncertainties because the ordinary fuzzy controllers (type-1 fuzzy controllers) have limited capabilities to directly handle data uncertainties [7]. There are five sources of the uncertainties in the type-1 fuzzy logic systems (T1-FLSs) [8,9]: (1) Uncertainties in the inputs to a FLS, which translate into uncertainties in the antecedents membership functions as the sensor measurements are affected by high noise levels from various sources. (2) Uncertainties in the control outputs, which translate into uncertainties in the consequents membership function of the FLS. (3) The meanings of the words that are used in the antecedents and consequents of rules can be uncertain (words mean different things to different people). (4) Uncertainties associated with the change in the operating conditions of the controller. Such uncertainties can translate into uncertainties in the antecedents and/or consequents' membership functions. (5) The data that used to tune the parameters of a T1-FLS may also be noisy. All of these uncertainties translate into uncertainties about fuzzy set membership functions. T1-FLS is not able to directly model such uncertainties because their membership functions are totally crisp.

On the other hand, the type-2 fuzzy sets (T2-FSs) that introduced by Zadeh in 1975 are able to model such uncertainties because their membership functions are themselves fuzzy; there are very useful in circumstances where it is difficult to determine an exact membership function of a fuzzy set [10]. The concept of the T2-FSs is an extension of the concept of the ordinary fuzzy sets (type-1 fuzzy sets; T1-FSs). A T2-FS is characterized by a fuzzy membership function (i.e., the membership grade for each element of this set is a fuzzy set in $[0, 1]$), unlike a T1-FS where the membership grade is a crisp number in $[0, 1]$ [10]. Therefore, a T2-FS provides additional degrees of freedom that make it possible to model and handle the uncertainties directly [11]. An IT2-FLC is a special case of a type-2 fuzzy logic system [9]. These are simpler to work with than general IT2-FLSs and distribute the uncertainty evenly among all admissible primary memberships [12]. The IT2-FLCs have been applied to various fields with great success [13–27].

The inverted pendulum on a cart system has the property of unstable, higher order, uncertain and highly coupled, which can be treated as a typical nonlinear control problem [28]. It provides an excellent experimental platform to test various control theories and techniques. Furthermore, an inverted pendulum system may also simulate many phenomena in the nature, such as walking robots, flying objects in space, and missile guidance [29]. The main problem in an inverted pendulum system is the uncertainty which is divided into two groups; parameter uncertainty and neglected linear, nonlinear and unmodelled dynamic uncertainty [30]. The parameter uncertainties can be caused by the parameters which are difficult or impossible to get a precise measure or that the parameters tend to vary as a function of time, temperature etc. For this pendulum system, the mean uncertain parameters are the Coulomb friction constants. With respect to neglected dynamics, the pendulum system does have unmodelled dynamics like bearings and track inclination. Further, the system includes also nonlinear elements dynamically. The nonlinear dynamic appears from the special the *sine* and *cosine* functions in the nonlinear model. Therefore, these uncertainties transmitted to the controller. So, the main objective of controlling the inverted pendulum is to minimize the effect of the system uncertainties. There are variety methods for an inverted pendulum control that are presented since now. The presented methods for an inverted pendulum control are divided generally into three groups. Classical methods such as PID controllers [31], modern methods such as an optimal control [32], and artificial intelligence methods such as neural networks and fuzzy control [33–37].

As reported in [38], a T1F-PD controller has been proposed for controlling an inverted pendulum system. Such a fuzzy PD structure is simple and can be theoretically analyzed. However, the main drawback of a T1F-PD controller is its limited capability to directly handle data uncertainties. Thus, the main objective of this paper is to develop an IT2F-PD controller taking into its consideration the advantages of the type-2 FLSs to overcome the uncertainty problems. The proposed IT2F-PD controller which contains two inputs (i.e., The error signal and the derivative of error signal) and one output (i.e., The control signal) has ability to minimize the effect of the structure uncertainties. The proposed controller is used for controlling the nonlinear inverted pendulum on a cart system to overcome the uncertainty problem and the effect of the external disturbances. The results are compared with other related controllers to test the robustness of the proposed controller to provide some improvements in performance over the related controllers under the effect of the system uncertainties and the external disturbances. The rest of this paper is organized as follows. In Section 2, the IT2F-PD controller is included. The description of the mathematical model of the nonlinear inverted pendulum is presented in Section 3. Section 4, presents the simulation results followed by the conclusions and the relevant references.

2. Interval type-2 fuzzy PD controller

The main objective of the controller design is to achieve better control performance in terms of stability and robustness for the system uncertainties and the environmental disturbances. Fig. 1, shows the structure of the IT2F-PD controller. It uses two input variables, i.e., $e(k)$ and $\Delta e(k)$, and one output variable, i.e., $u(k)$. Two scaling factors G_e and $G_{\Delta e}$ are employed to scale $e(k)$ and $\Delta e(k)$, respectively, as follows:

$$\begin{aligned} E(k) &= G_e e(k) = G_e(y_r(k) - y(k)) \\ \Delta E(k) &= G_{\Delta e}\Delta e(k) = G_{\Delta e}(e(k) - e(k-1)) \end{aligned} \tag{1}$$

where $y_r(k)$ is the system output reference signal, $y(k)$ is the output of the system under control, and k is the sampling instance. The output variable $u(k)$ and its scaling factor G_u are given by the following equation:

$$u(k) = G_u U(k) \tag{2}$$

where $U(k)$ is the output of the IT2F-PD controller.

The IT2F-PD controller works as follows: The crisp input from the input variables are first fuzzified into input IT2-FSs. The input IT2-FSs then activate the inference engine and the rule base to produce output IT2-FSs. The IT2-FLS rules will remain the same as in a T1-FLS, but the antecedents