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Manifestation of the magnetic moments of Cooper pairs in low-temperature properties of superconducting thin-film rings

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ABSTRACT

We argue that the intrinsic magnetic moments of the Cooper pairs can be detected in experiments with superconducting thin-film rings. At sufficiently low temperatures the magnetic field generated by the supercurrent, can cause the ordering of these magnetic moments. This magnetization of the superconductor produces changes in the supercurrent and magnetic induction distributions, the heat capacity and magnetic moment of the ring. It is shown how the intrinsic magnetic moment of the Cooper pairs can be extracted from low-temperature behaviors of these measurable quantities of the current-carrying rings made of the cuprate superconductors. Experimental determination of the magnetic moment of the Cooper pairs can shed light on the pairing symmetry in the HTC cuprates.

1. Introduction

The pairing symmetry in the cuprate superconductors is still controversial topic [1,2]. From the theoretical point of view, the *d*-wave pairing is mostly argued [3], though another spin-singlet channels, such as the *s*-wave pairing and admixed $d_{x^2-y^2}+s$ pair state are also discussed [1,2,4,5]. As noted in [6], the central symmetry of the CuO_2 planes can be broken down because of asymmetric surroundings of these planes in the parent compounds, as occurs in $YBa_2Cu_3O_{7-\delta}$. Also, doping-induced disorder can lead to the absence of this symmetry in the doped cuprates [7]. As a result, the mixed singlet–triplet state in the cuprates may be, in principle, possible [7–9].

High hopes to solve the problem of pairing symmetry are entrusted to the bulk- and phase-sensitive experimental techniques based on the macroscopic quantum coherent effects in superconductors. Though the $d_{x^2-y^2}$ pair state have been actively sought in numerous studies [1,2,10,11], a variety of experimental tests yielded conflicting results [12–16]. Therefore, another way for determining this symmetry is seemed to be urgent.

Here we focus on the well-known aspect that the pairing symmetry in superconductors and the intrinsic magnetic moment of the Cooper pairs are interrelated. So, for the *s*-wave pairing predicted by the Bardeen–Cooper–Schrieffer theory, the relative orbital angular momentum of the electron pairs is $l = 0$, and their orbital and spin magnetic moments are equal to zero. The most common opinion is that the cuprates inherent singlet *d*-wave pairing. This means that the orbital angular momentum of the electron pairs $l = 2$, and, respectively, the pairs have the orbital magnetic moment. For the mixed singlet–triplet state, besides the intrinsic orbital magnetic moment, the Cooper pairs have the spin magnetic moment as well.

Thus, the experimental determination of the magnetic moment of the Cooper pairs in superconductors and, in particular, the cuprates can shed light on the pairing symmetry. In addition, it is important to establish whether this symmetry is the same in different regions of the phase diagram of doped cuprates. For this purpose it is necessary to find experimental conditions in which the magnetic moments of the Cooper pairs will manifest themselves in observables.

In this paper we show that the magnetic moments of the Cooper pairs can be determined in experiments with thin-film rings made of the cuprate superconductors. The magnetic field generated by the persistent supercurrent in the ring, penetrates into the superconductor, if the ring thickness is less than the London penetration depth. In the case of sufficiently low temperatures this magnetic field causes the ordering of the magnetic moments of the Cooper pairs that leads to the local magnetization of the superconductor. This paramagnetic response produces changes in low-temperature properties of the ring such as the supercurrent and magnetic field distributions, the heat capacity and magnetic moment of the superconducting ring. The main result of our study is that from experimental measurements of the low temperature dependences of these observables one can determine the magnetic moment of the Cooper pairs.

2. Superconducting ring

Consider a flat thin-film ring made of the cuprate superconductor, with the inner radius *a*, the outer radius *b* and the ring thick-

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ness d. In the cylindrical coordinates used below, the z-axis coincides with the crystallographic c-axis, and the ring occupies the region $a \leqslant \rho \leqslant b$ and $-d/2 \leqslant z \leqslant d/2$. In this geometry the currents flow in the CuO_2 planes, and the current and magnetic field distributions in the ring are determined by the penetration depth λ_{ab}.

It is assumed that the magnetic field generated by the supercurrent in the ring, is less than the first critical field for the superconductor. This imposes a limit on the number of fluxoids trapped into the ring.

Calculations are carried out for the rings with $a \gg \lambda_{ab}$, $b - a \gg \lambda_{ab}$ and $d < \lambda_{ab}$. The latter allows to neglect the z-dependence of the current density and magnetic field into the superconducting film. Because of the circular symmetry, the current density and vector-potential have only a φ-component, $\mathbf{j}(\rho) = j(\rho)\mathbf{i}_\varphi$ and $\mathbf{A}(\rho) = A(\rho)\mathbf{i}_\varphi$, where $\mathbf{i}_\varphi$ is the azimuthal unit vector. The magnetic induction in the film and ring hole has only a z-component, $\mathbf{B}(\mathbf{r}) = B(\rho)\mathbf{i}_z$.

In the case of the d-wave symmetry, the Cooper pair has the intrinsic orbital magnetic moment $\boldsymbol{\mu}_i$. Its z-projection is

$$\mu_z = \mu_C m_L, \tag{1}$$

where μ_C is the order of the Bohr magneton and $m_L = 0, \pm 1, \pm 2$.

For the isotropic pairing the Cooper pair has no intrinsic magnetic moment.

3. Closed system of equations

As is well known, the cuprates exhibit coherent lengths in the nanometer range. The magnetic field in the ring does not change on such lengths. Hence, for description of the Zeeman energy of the Cooper pairs and their interaction between each other we can use their center-of-mass radius-vectors. The density matrix can be presented as:

$$\rho = e^{-(W_Z+W_d)/kT}/Sp(e^{-(W_Z+W_d)/kT}), \tag{2}$$

where T is the ring temperature, W_Z is the Zeeman energy,

$$W_Z = -\sum_i \boldsymbol{\mu}_i \mathbf{B}(\mathbf{r}_i), \tag{3}$$

and W_d is the energy of the magnetic dipole interaction of the Cooper pairs,

$$W_d = \frac{\mu_0}{2}\sum_{i\neq j}\left[\frac{\boldsymbol{\mu}_i\boldsymbol{\mu}_j}{|\mathbf{r}_i-\mathbf{r}_j|^3} - \frac{3(\boldsymbol{\mu}_i(\mathbf{r}_i-\mathbf{r}_j))(\boldsymbol{\mu}_j(\mathbf{r}_i-\mathbf{r}_j))}{|\mathbf{r}_i-\mathbf{r}_j|^5}\right]. \tag{4}$$

Here $\mathbf{r}_i$ is the center-of-mass radius-vector of i-electron pair.

Using (2)–(4), methods were developed to calculate the average $A = Sp(\rho\widehat{A})$ of an operator A in form of a power series in T^{-1} (see [17] and references therein). However, with respect to our problem, for not too low temperatures the energy (4) can be neglected as compared to (3). Evaluation of these temperatures is carried out in Section 4. Considering that in a real situation $T(<T_c) \gg \mu_C B(\rho = a)/k$, from (1)–(3) we obtain the average magnetic moment of the Cooper pair:

$$\mu_z(\mathbf{r}_i) = Sp(\rho\hat{\mu}_z) = \frac{2\mu_C^2 B(\mathbf{r}_i)}{kT}. \tag{5}$$

This ordering of the magnetic moments of Cooper pairs leads to the magnetization vector $\mathbf{J}(\rho) = J_z(\rho)\mathbf{i}_z$ in the ring. Using (5), we obtain:

$$J_z = \frac{2\mu_C^2 n_C B(\rho)}{kT}, \tag{6}$$

where n_C is the density of Cooper pairs.

From (6) we find the relationship between the magnetic induction and magnetic field strength:

$$\mathbf{B} = \mu_0\frac{T}{T-T_0}\mathbf{H}, \tag{7}$$

where $T > T_0$, and T_0 is the characteristic temperature, which depends on the intrinsic magnetic moment of the Cooper pairs:

$$T_0 = \frac{2\mu_0\mu_C^2 n_C}{k}. \tag{8}$$

In the derivation of the characteristic temperature (8) we neglected the energy of the magnetic dipole interaction of the Cooper pairs, which is proportional to $(T_0/T)^2$, as shown in Section 4. At $T \approx T_0$, this energy is the same order of magnitude as the Zeeman energy for all the Cooper pairs. Therefore, further we assume that the ring temperature $T \gg T_0$.

Now let us estimate T_0. For $\mu_C = \mu_B$ and the density of the Cooper pairs $n_C = 10^{20}\ \mathrm{cm}^{-3}$ we obtain $T_0 = 1.6$ mK. Of course, this characteristic temperature is too low compared with the superconducting transition temperature of the cuprates. Therefore there is a region of the ring temperature $T_0 \ll T \ll T_c$ in which all properties of the superconductor such as the penetration depth and density of the Cooper pairs, can be considered as constant.

From the Maxwell equation $rot\mathbf{H} = \mathbf{j}$ and (7), we obtain:

$$rot\mathbf{B} = \mu_0\frac{T}{T-T_0}\mathbf{j}. \tag{9}$$

Introducing $\mathbf{B} = rot\mathbf{A}_j$, where $\mathbf{A}_j$ is the vector potential generated by the supercurrent, from (9) we have:

$$\mathbf{A}_j(\mathbf{r}) = \frac{\mu_0 T}{4\pi(T-T_0)}\int\frac{\mathbf{j}(\mathbf{r}_1)}{|\mathbf{r}-\mathbf{r}_1|}d\mathbf{r}_1. \tag{10}$$

Relationship between the current density and total vector potential in the ring is given by the London equation:

$$\mathbf{j} = \frac{\Phi_0 n}{2\pi\mu_0\lambda_{ab}^2\rho}\mathbf{i}_\varphi - \frac{1}{\mu_0\lambda_{ab}^2}(\mathbf{A}_j + \mathbf{A}_L), \tag{11}$$

where $\Phi_0 = \pi\hbar/e$ is the fluxoid, n is the number of fluxoids trapped into the ring.

Unlike the vector-potential $\mathbf{A}_j$ generated by the Cooper pairs at their regular circular motion in the supercurrent states, the vector-potential $\mathbf{A}_L(\mathbf{r})$ is created by the intrinsic magnetic moments of the Cooper pairs. The vector potential at the point $\mathbf{r}$ generated by the magnetic moment of the i-Cooper pair, is:

$$\widehat{\mathbf{A}}_i(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{[\hat{\boldsymbol{\mu}}_i, \mathbf{r}-\mathbf{r}_i]}{|\mathbf{r}-\mathbf{r}_i|^3}, \tag{12}$$

where $\hat{\boldsymbol{\mu}}_i$ is the operator of the magnetic moment of the pair. The total vector potential is $\mathbf{A}_L = \sum_i \mathbf{A}_i(\mathbf{r})$, where the summation is taken over all the Cooper pairs. Using (2), (3) and (12), we find the average value of the total vector potential:

$$\mathbf{A}_L = A_L(\rho)\mathbf{i}_\varphi = \frac{1}{4\pi}\frac{T_0}{T}\int\frac{[\mathbf{B}(\mathbf{r}_1), \mathbf{r}-\mathbf{r}_1]}{|\mathbf{r}-\mathbf{r}_1|^3}d\mathbf{r}_1, \tag{13}$$

where the integration is carried out over the ring volume.

Thus, we have the closed system of two equations. Considering (10), (11) and (13), the first equation is:

$$\mathbf{j}(\mathbf{r},T) = \frac{\Phi_0 n}{2\pi\mu_0\lambda_{ab}^2\rho}\mathbf{i}_\varphi - \frac{T}{4\pi\lambda_{ab}^2(T-T_0)}\int\frac{\mathbf{j}(\mathbf{r}_1)}{|\mathbf{r}-\mathbf{r}_1|}d\mathbf{r}_1 - \frac{1}{4\pi\mu_0\lambda_{ab}^2}\frac{T_0}{T}\int\frac{[\mathbf{B}(\mathbf{r}_1), \mathbf{r}-\mathbf{r}_1]}{|\mathbf{r}-\mathbf{r}_1|^3}d\mathbf{r}_1, \tag{14}$$

and the second one is given by:

$$\mathbf{B}(\mathbf{r},T) = \frac{\mu_0 T}{4\pi(T-T_0)}\int\frac{[\mathbf{j}(\mathbf{r}_1), \mathbf{r}-\mathbf{r}_1]}{|\mathbf{r}-\mathbf{r}_1|^3}d\mathbf{r}_1. \tag{15}$$