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Inducing and detecting geometric phases with superconducting quantum circuits

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ABSTRACT

We propose a theoretically feasible scheme for controllably inducing and effectively detecting geometric phases by a superconducting circuit device. Only by adjusting the microwave pulses applied to the considered circuit, the non-Abelian and Abelian geometric phases can be induced controllably. Through considering the population difference after two composite evolutions, the noncommutative or commutative features of geometric phases can be explicitly shown. We address the scenarios of physical implementations with the available technology. Thus the scheme may offer a potential approach for synthetically investigating geometric phases with Josephson circuits.

1. Introduction

In terms of the initial work of Berry, the Abelian geometric phase can be acquired when a system Hamiltonian undergoes an adiabatic evolution [1]. Generalizing Berry phase to the case of degenerated states, Wilczek and Zee showed that the non-Abelian geometric phase emerges when the adiabatic evolution involves a twofold degenerate eigenspace [2]. Geometric phases play an important role in understanding novel phenomena associated with atomic physics and condensed matter physics [3]. During the past few years, geometric phases have attracted considerable attentions, mainly due to their potential applications in quantum computing [4–8], Cooper-pair pump [9,10], quantum phase transition [11], etc. To utilize geometric phases well, inducing geometric phases and detecting their features are crucial issues, and thus some attractive schemes have been put forward theoretically and experimentally [12–17]. Very recently, the non-Abelian adiabatic phases in a trapped ion were demonstrated effectively [18].

As artificial atom systems, superconducting nanocircuits have offered an excellent testing ground for the fundamental laws of quantum mechanics [19–23]. This is primarily because superconducting Josephson devices have some distinctive advantages. It is convenient to control the systems by adjusting the electromagnetic parameters such as gate voltages and bias fluxes [24,25]. The energy-level spacings are in the range of microwaves, and the controllable interactions between the artificial atoms and microwave fields can be realized optically [20]. The quantum-state measurements can be implemented accurately with current techniques as well [24]. Many valuable proposals to study geometric phases have been reported with Josephson nanocircuits.

The effective methods to detect Berry phases in superconducting circuits were presented in [26,27], where the geometric phases can be related to the probability amplitudes of quantum states. By adiabatically adjusting the biased microwave fields, the Berry phases were observed in a Josephson qubit experimentally [28]. With a device of superconducting charge pump, the Berry phase accumulated during each adiabatic pumping cycle can be determined quantitatively [29]. Moreover, utilizing a driven three-level transmon-type system, Berger et al. considered the effect of the second excited state on the Berry phase [30]. With the degenerated eigenstates of Josphson circuits, the non-Abelian geometric phases have been analyzed by adiabatic charge dynamics [31]. These previous works mainly focused on the Abelian or non-Abelian phases in adiabatic cases. However, how to induce controllably non-Abelian and Abelian types of geometric phases and to effectively detect their features are desirable to coherently control over quantum systems.

Motivated by the above purpose, we theoretically present a feasible scheme for inducing and detecting the geometric phases with the same superconducting circuit device. Since the four lowest levels of the considered circuit are well separated from the higher ones, they can be chosen as a four-level system. By adiabatically applying microwave pulses to the system, the non-Abelian and Abelian geometric phases can be controllably obtained in the dark-state space. We consider the population difference regarding the same quantum state after two composite evolutions, demonstrating explicitly the noncommutative or commutative features

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of the geometric phases. Based on the accessibly experimental technology, we further deal with the physical realizations of the proposed protocol. The scheme may provide us the significant step toward studying the geometric phases with Josephson nanocircuits.

The paper is organized as follows. In Section 2, we present a superconducting quantum circuit as an effective four-level system. The geometric phases are induced controllably in Section 3. Detecting the noncommutative or commutative features of geometric phases are given in Section 4. We address the physical realizations of detecting geometric phases in Section 5. Finally, discussion and conclusions are drawn in Section 6.

2. An effective four-level system

As schematically shown in Fig. 1, consider a superconducting Josephson device nicknamed quantronium circuit [32]. The basic Cooper-pair box consists of a low-capacitance island, connected to a superconducting reservoir through two Josephson junctions with identical capacitance C_J and Josephson energy E_J. The island has excess Cooper-pairs n. Through the gate capacitance C_g, the island can be biased by voltage sources containing static V_d and ac $\widetilde{V}_j$ components. Another Josephson junction with coupling energy $E_{J0} \gg E_J$ is inserted in the circuit loop. By applying a current pulse $I_b(t)$ to E_{J0} and monitoring the voltage $U(t)$, the desired quantum states can be read out effectively [32].

A static voltage V_d applied to the gate capacitance C_g induces offset charge number $n_d = C_gV_d/2e$. The Hamiltonian describing the static system reads $H_0 = E_c(n - n_d)^2 - \overline{E}_J \cos\theta$, in which θ and n fulfils the conjugate relation, $[\theta, n] = i$. The characteristic charging energy is $E_c = 2e^2/C_\Sigma$, with $C_\Sigma = (C_g + 2C_J)$ being the total capacitance of the box. And the effective Josephson energy is $\overline{E}_J = 2E_J\cos\left(\frac{\delta}{2}\right)$, δ indicates the phase difference across the combination of the two junctions. In our considered regime, $\overline{E}_J$ has the same order of magnitude as E_c. Within the Cooper-pair basis $\{|n\rangle, |n+1\rangle\}$, the Hamiltonian is rewritten as [33,34]

$$H_0 = \sum_n [E_c(n - n_d)^2 |n\rangle\langle n| - \frac{\overline{E}_J}{2}(|n\rangle\langle n+1| + H.c.)]. \tag{1}$$

According to Eq. (1), the first four eigenlevels E_k as functions of gate charge n_d for a selected $\overline{E}_J = 2.5E_c$ are given in Fig. 2, with k = 1, 2, 3 and 4. Under the radiations from the external fields, the system can serve as an effective four-level system. The four levels are denoted by $|s_k\rangle$. And each level state is the superposition of many Cooper-pair states [35], $|s_k\rangle = \sum_n C_{kn} | n\rangle$, where $C_{kn} = \langle n|s_k\rangle$ are the superposition coefficients and satisfy $\sum_n |C_{kn}|^2 = 1$.

The system working at the magic point ($n_d = 0.5$) is resilient against the dephasing effects [32]. At the bias point $n_d = 0.5$, we have the eigenfunctions $|s_2\rangle$ and $|s_4\rangle$ of the Hamiltonian H_0, see Fig. 3(a). The transition $|s_2\rangle \leftrightarrow |s_4\rangle$ induced by the external field is required in our scheme. However, the selection rule determined by the parity symmetry of eigenfunctions [36] impedes the desired transition between $|s_2\rangle$ and $|s_4\rangle$, as will be mentioned below. So, n_d must be slightly away from *0.5* to break the parity symmetry. Here the working point is chosen as $n_d = 0.45$ [37]. We obtain the eigenfunctions $|s_j\rangle$ and $|s_4\rangle$ plotted in Fig. 3(b,c and d), with j = 1, 2 and 3. As a consequence, allowed by the selection rules, the level transitions $|s_j\rangle \leftrightarrow |s_4\rangle$ can be implemented via microwave pulses $\widetilde{V}_j$. Meanwhile, the system that is biased near the magic point keeps the robust coherence as much as possible.

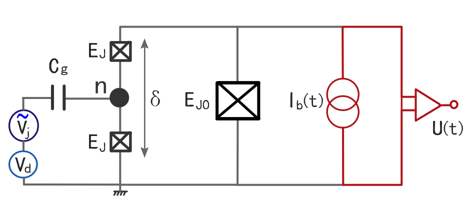

Fig. 1. Schematic diagram of the considered circuit device, this figure is reproduced from Fig. 1 of Ref. [32].

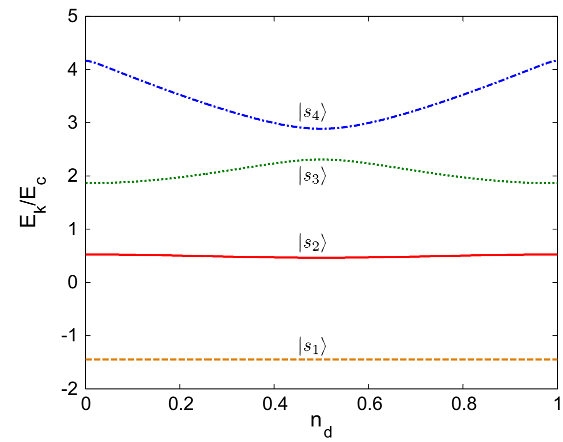

Fig. 2. The first four levels E_k as functions of gate charges n_d, and energies are given in units of E_c. The considered level states are denoted by $|s_k\rangle$, k = 1, 2, 3 and 4.

3. Controllable inducements of geometric phases

To obtain the level transitions between $|s_j\rangle$ and $|s_4\rangle$, we apply ac pulse voltages $\widetilde{V}_j = V_j(t)\cos(\omega_j t)$ to the gate capacitance, where $V_j(t)$ are small time-dependent magnitudes obeying $C_gV_j(t)/2e \ll 1/2$, and ω_j are ac frequencies. Since the microwave pulses are diagonally coupled to the charge states, the interaction Hamiltonians are given by [37] $\widetilde{H}_j = -2E_c\tilde{n}_j(t)\sum_n(n - n_d) | n\rangle\langle n |$, where $\tilde{n}_j(t) = n_j(t)\cos(\omega_j t)$, with $n_j(t) = C_gV_j(t)/2e$ being the reduced amplitude. The fast oscillating terms such as $\tilde{n}_j^2(t)$ have been eliminated under the rotating wave approximation. The transitions can be described by matrix elements $t_{j4} = \langle s_j | \widetilde{H}_j | s_4\rangle$ that are induced by microwave pulses $\tilde{n}_j(t)$. Here the applied ac frequencies ω_j are resonantly matched with the transition frequencies $\omega_{4j} = (E_4 - E_j)/\hbar$. Therefore, the other microwave-induced transitions such as $|s_1\rangle \leftrightarrow |s_3\rangle$ and $|s_2\rangle \leftrightarrow |s_3\rangle$ can all be neglected safely [38].

As stated above, we have obtained the n_d-dependent probability amplitudes c_{jn} and c_{4n} with respect to $|n\rangle$, see Fig. 3(a–d). Now we analyze the transition matrix elements $t_{j4} = \langle s_j | \widetilde{H}_j | s_4\rangle$ to explain the prohibitions or allowances of the transitions $|s_j\rangle \leftrightarrow |s_4\rangle$. The matrix elements can be expressed as $t_{j4} = -2E_c\tilde{n}_j(t)O^{(n)}_{j4}$, where $O^{(n)}_{j4} = \sum_n(n - n_d)c^*_{jn}c_{4n}$ stand for n_d-dependent wavefunction overlaps between $|s_j\rangle$ and $|s_4\rangle$. It is numerically found that $O^{(n)}_{24} = 0$ for $n_d = 0.5$, namely, the transition between $|s_2\rangle$ and $|s_4\rangle$ is thus prohibited. Physically, there exists an even symmetry between $|s_2\rangle$ and $|s_4\rangle$, as illustrated in Fig. 3(a). Differently, by eliminating the even symmetry, we have $O^{(n)}_{24} = -0.1187$ for $n_d = 0.45$, which means that the parity symmetry-determined selection rule allows the transition between $|s_2\rangle$ and $|s_4\rangle$. Similarly, using the relevant parameters, we have $O^{(n)}_{14} = -0.0942$ and $O^{(n)}_{34} = 1.177$ for $n_d = 0.45$. Therefore, the transitions between $|s_j\rangle$ and $|s_4\rangle$ are all allowed at the chosen working point.

As sketched in Fig. 4(a), we have the level transitions $|s_j\rangle \leftrightarrow |s_4\rangle$ via $\tilde{n}_j(t)$, j = 1, 2 and 3. In the rotating wave approximation, the interaction Hamiltonian within the rotating frame is described by [12,38]

$$H^{(a)}_I = \hbar(\Omega_1|s_1\rangle\langle s_4| + \Omega_2|s_2\rangle\langle s_4| + \Omega_3|s_3\rangle\langle s_4|) + H.c., \tag{2}$$

with Rabi frequencies $\Omega_j = n_j(t)E_c \mid O^{(n)}_{j4} \mid /\hbar$. Note that Ω_j can be switched on and off effectively by adjusting $\tilde{n}_j(t)$. The couplings between $|s_j\rangle$ and $|s_4\rangle$ will exist when $\tilde{n}_j(t)$ are turn on. Otherwise,