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Stress and magnetostriction in an infinite hollow superconducting cylinder with a filling in its central hole

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ABSTRACT

In this paper, the stress and magnetostriction induced by flux pinning in an infinite hollow cylinder of type-II superconductor with a non-superconductive filling in its central hole are analytically investigated. Based on the exponent model of critical state of superconductor, the magnetic and current distributions in the hollow cylinder superconductor are obtained firstly. The stress and magnetostriction of the composite superconductive cylinder are then formulated and the magnetoelastic behaviors are characterized analytically. The results show that the hoop stress concentration near the central hole is dominant due to the tension Lorentz force and it is certainly suppressed by filling a non-superconductive material in the hole. Without change of the magnetization characteristic of the superconductor, the filling material provides effective way to remedy the stress state at the verge of the hollow in the superconductive cylinder by adjusting its Young modulus. The magnetostriction of the composite cylinder under the cycled magnetic field is further presented. Effect of the applied maximum field, complete penetration field parameter and filling material parameter on the profile of the cycled magnetostriction for the composite superconducting cylinder is discussed in detail.

1. Introduction

With the characteristics of trapping high magnetic field and bearing large critical current density, bulk high temperature superconductors (HTS's) have been widely used in engineering, such as magnetic levitation trains, magnetic bearings, magnetic separators, hysteresis motors and generators, high temperature superconductor undulator, resonant X-ray scattering, and flywheel energy storage [1–5]. In order to pursue larger critical current density in HTS's, plenty of experimental researches on different compound superconductive material or large single-grain structures have been carried out to improve the materials' electromagnetism properties [6,7]. Meanwhile, the theoretical studies on the critical current density and the flux line distribution were developed. Brandt et al. analyzed the current density, magnetic field and magnetic moment in a type-II superconductor when it carried transport current or it was placed in a magnetic field [8–11]. Zeldov et al. investigated the magnetization and transport currents in thin superconducting films and slabs to capture the critical-state behavior of the superconductor theoretically for arbitrary sequence of applied transport currents and perpendicular magnetic field [12]. The hysteresis losses in hard superconductors were also reported by Norris [13,14].

When the hard superconductor trap magnetic field, the current line appear in the superconductor and a large electromagnetic force which sometimes leads to a fatal fracture is exerted and threats the application of the superconductors. Johansen [15–19] systematically investigated the stresses, strains and magnetostrictions in different shaped type-II superconductors under magnetic field or with transport current, and indicated that the research on the mechanical instabilities of superconductors are even of more crucial important than those on the critical current density. It was revealed that the bulk superconductor may be tensioned by the electromagnetic forces with the applied magnetic field decreasing, and the large forces even lead to a crack generation or crack propagation in the material [20]. Zeng et al. [21] studied the stress distribution in a hollow cylindrical superconductor based on the FEM simulation to evaluate the stress concentration values at the verge of the hollow in the superconductive cylinder. In the practical engineering applications, there are unavoidable flaws and microcracks emerging accompanying with the preparation of hard superconducting materials, and even some structures need a hole in superconductors. Therefore, an effective way to suppress the stress concentration near the hole or cracks comes to heal as a vital problem. Fuchs et al. [22] reported that the Ag addition bulk YBCO would trap higher magnetic field with a bandage. Tomi-

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ta and Murakami [23] further found that, by the way of penetrating the epoxy resin into microcracks and macrocracks of the bulk superconductor, the corresponding mechanical properties can be improved which results in the enhancement of the bulk's field-trapping capability.

In this article, we present an analysis on the stress and magnetostriction characteristics of an infinite length superconductive cylinder with a center hole filling with non-superconductive materials. The stress and magnetostriction of the filling superconductive cylinder were formulated and the magnetoelastic behaviors have been characterized analytically. Without changing of the superconductive cylinder's magnetization properties, the filling in the hollow cylinder can effectively prevent stress concentration emerging at the verge of the central hole in the cylinder. The magnetostriction profile of the cylinder under different cycled field is presented in details.

2. Formulation

Consider a type-II superconductive circular cylinder of infinite length filling with non-superconductive materials in its central hole. It is placed in a magnetic field B_a oriented parallel to the cylinder axis denoted by z, as sketched in Fig. 1. The outer and inner radii of the cylinder are respectively R and a. A flux pinning force $F_p(r)$ is generated in the cylinder from the applied field and the induced hoop current J and magnetic flux density appear in the cylinder. Based on the critical state assumption, the flux pinning force is given, with the value being equal to this Lorentz force F_L, in the form

$$F_p(r) = F_L = -\frac{1}{2\mu_0}\frac{d}{dr}(B^2), \tag{1}$$

in which B is the magnetic flux density in the superconductor cylinder. It is showed that, in literature [24], the exponential model has the advantage on capturing the magnetostriction curve feature in a magnetic field. In this paper, the exponent model $J_c(B) = \mp J_{c_0} \exp(-|B|/B_0)$ is adopted to describe the current density behavior of the superconductor, in which J_{c_0} and B_0 are constants. It is not difficult to obtain the distribution of current density and flux density in the cylinder in the framework of the critical state model, and the detailed derivations and expressions of them, which are similar to ones in the literature [24], will be not presented and discussed herein. Next, the mechanical properties are discussed analytically.

The assumption of both the superconductor phase and the filling material phase being the tetragonal lattice is adopted which follows a way by Yong and Zhou [25] and Ceniga and Diko [26]

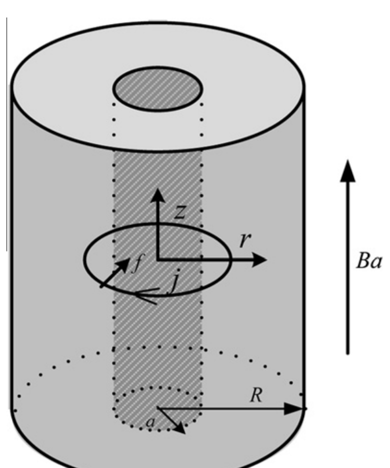

Fig. 1. Rough sketch of a superconductive hollow cylinder with filling non-superconductive material in its central hole being placed in a magnetic field B_a.

studied for superconductor composites. The relations between stresses and strains are written in the forms as follows:

$$\begin{aligned}
\varepsilon_r^\alpha &= s_{11}^\alpha\sigma_r^\alpha + s_{12}^\alpha\sigma_\varphi^\alpha + s_{13}^\alpha\sigma_z^\alpha \\
\varepsilon_\varphi^\alpha &= s_{12}^\alpha\sigma_r^\alpha + s_{11}^\alpha\sigma_\varphi^\alpha + s_{13}^\alpha\sigma_z^\alpha \\
\varepsilon_z^\alpha &= s_{31}^\alpha\sigma_r^\alpha + s_{31}^\alpha\sigma_\varphi^\alpha + s_{33}^\alpha\sigma_z^\alpha,
\end{aligned} \tag{2}$$

where $s_{ij}^\alpha = \frac{\delta_{ij}-\mu_j^\alpha(1-\delta_{ij})}{E_i^\alpha}$, $(i,j = 1,2,3)$, are compliance coefficients, δ_{ij} means the Kronecker's delta and it takes 1 for $i = j$ and 0 for $i \neq j$. The superscripts, $\alpha = m, f$, respectively represents quantities in the superconductive hollow cylinder ($\alpha = m$) and those in the no-superconductive filling material ($\alpha = f$); the parameter E_i^α and μ_j^α are respectively the Young modulus and Poisson's ratio. In consideration of the infinite length cylinder in the z direction, there is $\varepsilon_z^\alpha = 0$. Therefore the stresses in Eq. (2) are calculated as:

$$\begin{aligned}
\sigma_r^\alpha &= \frac{\prod_2^\alpha}{\prod_1^\alpha}\varepsilon_r^\alpha + \frac{\prod_3^\alpha}{\prod_1^\alpha}\varepsilon_\varphi^\alpha \\
\sigma_\varphi^\alpha &= \frac{\prod_3^\alpha}{\prod_1^\alpha}\varepsilon_r^\alpha + \frac{\prod_2^\alpha}{\prod_1^\alpha}\varepsilon_\varphi^\alpha, \quad (\alpha = m,f) \\
\sigma_z^\alpha &= \frac{(s_{12}^\alpha s_{23}^\alpha - s_{13}^\alpha s_{22}^\alpha)}{\prod_1^\alpha}\varepsilon_r^\alpha + \frac{(s_{13}^\alpha s_{21}^\alpha - s_{11}^\alpha s_{23}^\alpha)}{\prod_1^\alpha}\varepsilon_\varphi^\alpha.
\end{aligned} \tag{3}$$

Here, $\prod_1^\alpha = (s_{11}^\alpha - s_{12}^\alpha)(-2s_{13}^\alpha s_{31}^\alpha + s_{11}^\alpha s_{33}^\alpha + s_{12}^\alpha s_{33}^\alpha)$, $\prod_2^\alpha = s_{11}^\alpha s_{33}^\alpha - s_{13}^\alpha s_{31}^\alpha$, and $\prod_3^\alpha = s_{13}^\alpha s_{31}^\alpha - s_{12}^\alpha s_{33}^\alpha$, $(\alpha = m, f)$. Due to the axial symmetry of the cylindrical structure and magnetic loading, we can write the strain components with the radial displacement as

$$\varepsilon_r^\alpha = \frac{du_r^\alpha}{r}, \quad \varepsilon_\varphi^\alpha = \frac{u_r^\alpha}{r}, \quad (\alpha = m,f). \tag{4}$$

According to the linear elastic theory, the shear stress in the symmetric double-layer cylinder is zero at this moment. The force balance equations in the state of plane strain, under the cylindrical coordinate system, are formulated as follows:

$$\frac{\partial\sigma_r^\alpha}{\partial r} + \frac{\sigma_r^\alpha - \sigma_\varphi^\alpha}{r} + \delta_\alpha F_p(r) = 0, \quad (\alpha = m,f). \tag{5}$$

in which σ_r is the axial stress and σ_φ is the stress along the azimuth angle, $\delta_\alpha = 1$ for $\alpha = m$ and $\delta_\alpha = 0$ for $\alpha = f$.

In combination of Eqs. (3)–(5), the governing equations are obtained as following forms

$$\frac{d^2u_r^\alpha}{dr^2} + \frac{1}{r}\frac{du_r^\alpha}{dr} - \frac{u_r^\alpha}{r^2} + \frac{\prod_1^\alpha}{\prod_2^\alpha}\delta_\alpha F_p(r) = 0, \quad (\alpha = m,f). \tag{6}$$

With the assumption of no-slip constrains on the interfaces between the filling material and the superconductor layer, the normal stress and displacement between the superconductor and filling material are continuous. The corresponding connecting and boundary conditions are given by

$$u_r^m\big|_{r=a} = u_r^f\big|_{r=a}, \quad \sigma_r^f\big|_{r=a} = \sigma_r^m\big|_{r=a}, \quad \sigma_r^m\big|_{r=R} = 0. \tag{7}$$

By solving the above governing differential equations, the stresses and displacements of the no-superconductive filling and superconductor cylinder can be obtained analytically as follows:

$$\begin{aligned}
\sigma_r^f &= \sigma_\varphi^f = \frac{\prod_2^f + \prod_3^f}{\prod_1^f}C_1^f \\
\sigma_r^m &= \frac{1}{2\mu_0}B^2 + \frac{1}{2\mu_0}\left(\frac{\prod_3^m}{\prod_2^m} - 1\right)\frac{1}{r^2}\int_a^r r'B^2dr' + \frac{\prod_3^m + \prod_2^m}{\prod_1^m}C_1^m + \frac{\prod_3^m - \prod_2^m}{\prod_1^m}\frac{1}{r^2}C_2^m \\
\sigma_\varphi^m &= \frac{1}{2\mu_0}\frac{\prod_3^m}{\prod_2^m}B^2 - \frac{1}{2\mu_0}\left(\frac{\prod_3^m}{\prod_2^m} - 1\right)\frac{1}{r^2}\int_a^r r'B^2dr' + \frac{\prod_3^m + \prod_2^m}{\prod_1^m}C_1^m - \frac{\prod_3^m - \prod_2^m}{\prod_1^m}\frac{1}{r^2}C_2^m
\end{aligned} \tag{8}$$