Alexandria Engineering Journal (2014) 53, 219–224

Alexandria University

Alexandria Engineering Journal

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ORIGINAL ARTICLE

Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet

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Received 1 August 2013; revised 4 November 2013; accepted 9 November 2013
Available online 5 December 2013

KEYWORDS

Three-dimensional;
Water-based nanofluid;
Exponentially stretching sheet;
Numerical study

Abstract The aim of the present study is to analyze the flow of three-dimensional water-based nanofluid over an exponentially stretching sheet. The transport equations are transformed into non-linear, coupled similarity equations using three-dimensional exponential type similarity transformations. These equations are solved numerically to obtain the velocities and temperature in the respective boundary layers. Results are presented to illustrate the effects of various parameters including the temperature exponent, stretching parameter and volume fraction of three different types of nanoparticles, such as copper (Cu), alumina (Al_2O_3) and titanium dioxide (TiO_2) with water as a base fluid.

1. Introduction

The studies of flows over a stretching sheet have gained admirable attention due to its extensive use in engineering applications, such as extrusion of polymer sheet from a dye or in the drawing of plastic films, bundle wrapping, hot rolling, extrusion of sheet material, wire rolling and glass fiber. Main applications related to such kind of motion are in the field of geophysical fluid dynamics that is the study of naturally occurring, large-scale flows on Earth and elsewhere, but mostly on Earth. Although the discipline encompasses the motions of both fluid phases – liquids (waters in the ocean, molten rock in the outer core) and gases (air in our atmosphere, atmospheres of other planets, ionized gases in stars) – a restriction is placed on the *scale* of these motions. Only the larger-scale motions fall within the scope of geophysical fluid dynamics. For example, problems related to river flow, microturbulence in the Upper Ocean, and convection in clouds are traditionally viewed as topics specific to hydrology, oceanography, and meteorology, respectively [1].

Sakiadis [2,3] pioneered the concept of boundary layer for continuously stretching surface. Study related to stretching sheet has concerned many articles [4–8] to investigate fluid behavior with various physical aspects. Crane [9] contributed appreciative work to extend the idea of Sakiadis. He presented the idea of both linear and exponentially stretching surface for steady boundary layer flow and found the exact similar

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

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http://dx.doi.org/10.1016/j.aej.2013.11.003

solutions in closed analytical form. Meanwhile, many authors modify the idea of stretching sheet for different fluid models for both nonlinear and exponentially stretching sheet [10–14].

Nanofluids are homogenous mixture of base fluid and nanoparticles. Suspended metallic or non-metallic particles change the transport properties and heat conduction characteristics of the common base fluids include water, organic liquids (e.g. ethylene, tri-ethylene–glycols, refrigerants), engine oils and lubricants, bio-fluids, polymeric solution and other common liquids. In fact, enhanced thermal conductivity is based on the nanoparticles. Nanoparticle is distributive part of any material whose diameter is about less than 100 nm. The effectiveness of heat transfer enhancement has been found to be dependent on the amount of dispersed particles, material type and particle shape. A number of well-known nanoparticles are created from different materials, such as oxide ceramics (Al_2O_3, CuO), metal nitrides (AlN, SiN), carbide ceramics (Sic, Tic), metals (Cu, Ag, Au), carbons in various (e.g., diamond, graphite, carbon nanotubes, fluorine) and functional nanoparticles. Initially, attempt of this classical model is introduced by Maxwell [15] for statistically homogenous, isotropic composite materials with randomly dispersed spherical particles. In the development of nanofluids, Choi [16] presented the concept of nanofluids for suspension of liquids containing ultra-fine particles (diameter less than 50 nm). In the past few years, many experimental investigations on the thermal conductivity of nanofluids have been studied. Yoo et al. [17] measured the thermal conductivity of nanofluids for TiO_2, Al_2O_3 and Fe and found that large enhancement in thermal conductivity compare to the base fluids. A phenomenon of nanofluid over a stretching sheet for laminar boundary layer flow was first presented by Khan and Pop [18]. In the recent developments, vast amount of the literature concern with the nanofluid for both Newtonian and non-Newtonian models is available but we refer few recent studies [19–30]. Nadeem and Lee [31] extended the idea of nanofluid over an exponentially stretching sheet. Recently, Bachok el al. [32] presented the steady three-dimensional stagnation point flow in a nanofluid.

Motivation of the present study is to extend the idea of Liu et al. [33] to investigate the flow of three-dimensional exponentially stretching sheet for nanofluid. In the present study we have assumed water as a base fluid and also consider the effects of nanoparticle volume fraction taken into the account. Moreover, we have considered three different types of nanoparticles such as copper (Cu), alumina (Al_2O_3), and titanium dioxide (TiO_2). Introducing the similar transformations, the momentum and energy equations, under boundary layer assumptions reduce the equations to the set of ordinary differential equations. Solutions of reduced coupled differential equations are solved numerically. Graphical results for emerging parameters are being described through graphs and discussion. Physical behaviors of reduced Sherwood and local Nusselt number have discussed through graphs.

2. Mathematical model

Consider three-dimensional (3D) steady boundary layer flow past a stretching sheet in the presence of nanoparticles, it is also considered that the sheet is stretched with the different velocities U_w, V_w along the Cartesian coordinate axis, x- and y-axes respectively whereas the fluid placed along the z direction is taken to be at rest. Moreover, we have considered the constant temperature T_w at wall and the ambient temperature T_∞. Under the above assumptions the governing continuity, momentum and energy equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\mu_{nf}\left(\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial z^2}\right) \tag{4}$$

subject to the boundary conditions

$$\left.\begin{array}{l} u=U_w,\quad v=V_w,\quad w=0,\quad T=T_w,\quad \text{at } z=0\\ u\to 0,\quad v\to 0,\quad T\to T_\infty,\quad \text{as } z\to\infty \end{array}\right\} \tag{5}$$

In above expressions u, v and w are the velocity components along the x, y and z-axes, respectively; T is the temperature, ρ_{nf} is the nanofluid density, μ_{nf} is the viscosity of nanofluid and α_{nf} is the thermal diffusivity of nanofluid defined as follows

$$\left.\begin{array}{l} \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\quad \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s\\ (\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s,\quad \nu_{nf}=\frac{\mu_{nf}}{\rho_{nf}}\\ \frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)},\quad \alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}} \end{array}\right\} \tag{6}$$

where k_{nf} is the thermal conductivity of the nanofluid, $(\rho c_p)_{nf}$ is the heat capacity of nanofluid and ϕ is the solid volume fraction of nanoparticles.

We look for a similarity solution of Eqs. (1)–(4) of the following form

$$\left.\begin{array}{l} u=U_0 e^{\frac{x+y}{L}}f'(\eta),\quad v=V_0 e^{\frac{x+y}{L}}g'(\eta),\\ w=\left(\frac{\nu U_0}{2L}\right)^{1/2} e^{\frac{x+y}{L}}\{f(\eta)+\eta f'(\eta)+g(\eta)+\eta g'(\eta)\},\\ T=T_\infty+T_0 e^{\frac{A(x+y)}{2L}}\theta(\eta),\quad \eta=\left(\frac{U_0}{2\nu L}\right)^{1/2} e^{\frac{x+y}{L}}z \end{array}\right\} \tag{7}$$

where U_0, V_0 and T_0 are constants, L is the reference length and A is the temperature exponent. Employing the similarity variables (7), Eqs. (1)–(5) reduces to the following nonlinear system of ordinary differential equations:

$$\left.\begin{array}{l} \frac{1}{(1-\phi)^{2.5}\{1-\phi+\phi(\rho_s/\rho_f)\}}f'''+(f+g)f''-2(f'+g')f'=0\\ \frac{1}{(1-\phi)^{2.5}\{1-\phi+\phi(\rho_s/\rho_f)\}}g'''+(f+g)g''-2(f'+g')g'=0\\ \frac{1}{\Pr}\frac{1}{(1-\phi)^{2.5}[1-\phi+\phi\{(\rho c_p)_s/(\rho c_p)_f\}]}\theta''+(f+g)\theta'-A(f'+g')\theta=0 \end{array}\right\} \tag{8}$$

subjected to the boundary conditions (5) which become

$$\left.\begin{array}{l} f(0)=0,\quad f'(0)=1,\quad g(0)=0,\quad g'(0)=\alpha,\quad \theta(0)=0\\ f'\to 0,\quad g'\to 0,\quad \theta\to 0,\ \text{as } \eta\to\infty \end{array}\right\} \tag{9}$$

Here, primes denote differentiation with respect to η, $\Pr=(\mu c_p)_f/k_f$ is the Prandtl number and $\alpha=V_0/U_0$ is the stretching ratio parameter.