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ORIGINAL ARTICLE

General traveling wave solutions of the strain wave equation in microstructured solids via the new approach of generalized (G'/G)-expansion method

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Abstract The new approach of generalized (G'/G)-expansion method is significant, powerful and straightforward mathematical tool for finding exact traveling wave solutions of nonlinear evolution equations (NLEEs) arise in the field of engineering, applied mathematics and physics. Dispersive effects due to microstructure of materials combined with nonlinearities give rise to solitary waves. In this article, the new approach of generalized (G'/G)-expansion method has been applied to construct general traveling wave solutions of the strain wave equation in microstructured solids. Abundant exact traveling wave solutions including solitons, kink, periodic and rational solutions have been found. These solutions might play important role in engineering fields.

1. Introduction

Microstructured materials like alloys, crystallites, ceramics, and functionally graded materials have gained wide application. The modeling of wave propagation in such materials should be able to account for various scales of microstructure [1–3]. The existence and emergence of solitary waves in complicated physical problems apart from the model equations of mathematical physics should be analyzed with sufficient correctness. It has recently become more attractive to obtain exact solutions of nonlinear partial differential equations through computer algebra that facilitate complex and tedious algebraic computations. Evaluating exact and numerical solutions, in particular, traveling wave solutions, of nonlinear equations in mathematical physics play an important role in soliton theory [4,5]. There are several studies where the governing equations for waves in microstructured solids have been derived and solitary waves are analyzed [6,7]. Mathematical modeling of physical and engineering problems is innately governed by nonlinear partial differential equations and hence investigation of exact solutions of nonlinear partial differential equations is

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very much important. The exact solutions of these equations give information about the structure of these problems. To this end, in the past several decades various effective methods have been developed and established to solve and understand the mechanisms of these phenomena. Among them the notable are, the modified simple equation method [8–12], the (G'/G)-expansion method [13–21], the Adomian decomposition method [22], the Lie group symmetry method [23], the homotopy analysis method [24,25], the first integration method [26], the inverse scattering method [27], the theta function method [28,29], the tanh-function method [30,31], the extended tanh-function method [32,33], the homogeneous balance method [34], the Jacobi elliptic function method [35,36], the Hirota's bilinear method [37], the sine-cosine method [38], etc.

Recently, Naher and Abdullah [39] presented an effective and straightforward method, called the new approach of generalized (G'/G) expansion method to obtain exact traveling wave solutions of NLEEs. In this article, we put forth the new approach of generalized (G'/G) expansion method to construct exact traveling wave solutions including solitons, kink, periodic and rational solutions to the strain wave equation in microstructured solids.

2. Description of the new generalized (G'/G)-expansion method

Consider the general nonlinear partial differential equation

$$H(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \ldots), \tag{1}$$

where $u = u(x,t)$ is an unknown function, H is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order partial derivatives and the nonlinear terms are involved. The main steps of the method are as follows:

Step 1: Combining the real variables x and t by a compound variable ξ, we suppose that

$$u(x,t) = u(\xi), \quad \xi = x \pm Vt, \tag{2}$$

where V is the speed of the traveling wave. The transformation (2) transforms Eq. (1) into an ordinary differential equation (ODE) for $u = u(\xi)$

$$F(u, u', u'', u''', \ldots), \tag{3}$$

where F is a function of $u(\xi)$ and its derivatives.

Step 2: According to possibility, Eq. (3) can be integrated term by term one or more times, yields constant(s) of integration. The integral constant may be zero, for simplicity.

Step 3: Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$u(\xi) = \sum_{i=0}^{N} d_i (p+M)^i + \sum_{i=1}^{N} e_i (p+M)^{-i}, \tag{4}$$

where either d_N or e_N may be zero, but both of them could be not zero at a time. d_i $(i = 0, 1, 2, \ldots, N)$ and e_i $(i = 1, 2, \ldots, N)$ and p are constants to be determined later and $M(\xi)$ is

$$M(\xi) = (G'/G) \tag{5}$$

where $G = G(\xi)$ satisfies the following auxiliary nonlinear ordinary differential equation:

$$h_1 G G'' - h_2 G G' - h_3 (G')^2 - h_4 G^2 = 0, \tag{6}$$

where the prime stands for derivative with respect to ξ; h_1, h_2, h_3 and h_4 are real parameters.

Step 4: To determine the positive integer N, taking the homogeneous balance between the highest order nonlinear terms and the derivatives of the highest order come out in Eq. (3).

Step 5: Substitute Eqs. (4) and (6) including Eq. (5) into Eq. (3) and the value of N obtained in Step 4, we obtain polynomials in $(p+M)^N$ $(N = 0, 1, 2, \ldots)$ and $(p+M)^{-N}$ $(N = 0, 1, 2, \ldots)$. Then, we collect each coefficient of the resulted polynomials to zero yields a set of algebraic equations for d_i $(i = 0, 1, 2, \ldots, N)$ and e_i $(i = 1, 2, \ldots, N)$, p and V.

Step 6: Suppose that the value of the constants d_i $(i = 0, 1, 2, \ldots, N)$, e_i $(i = 1, 2, \ldots, N)$, p and V can be found by solving the algebraic equations obtained in Step 5. Since the general solution of Eq. (6) is well known, inserting the values of d_i, e_i, p and V into Eq. (4), we obtain more general type and new exact traveling wave solutions of the nonlinear partial differential Eq. (1).

Using the general solution of Eq. (6), we have the following solutions of Eq. (5):

Family 1: When $h_2 \neq 0$, $\psi = h_1 - h_3$ and $\Phi = h_2^2 + 4h_4(h_1 - h_3) > 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{h_2}{2\psi} + \frac{\sqrt{\Phi}}{2\psi} \frac{A \sinh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right) + B \cosh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)}{A \cosh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right) + B \sinh\left(\frac{\sqrt{\Phi}}{2h_1}\xi\right)} \tag{7}$$

Family 2: When $h_2 \neq 0$, $\psi = h_1 - h_3$ and $\Phi = h_2^2 + 4h_4(h_1 - h_3) < 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{h_2}{2\psi} + \frac{\sqrt{-\Phi}}{2\psi} \frac{-A \sin\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right) + B \cos\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)}{A \cos\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right) + B \sin\left(\frac{\sqrt{-\Phi}}{2h_1}\xi\right)} \tag{8}$$

Family 3: When $h_2 \neq 0$, $\psi = h_1 - h_3$ and $\Phi = h_2^2 + 4h_4(h_1 - h_3) = 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{h_2}{2\psi} + \frac{C_2}{C_1 + C_2\xi} \tag{9}$$

Family 4: When $h_2 = 0$, $\psi = h_1 - h_3$ and $\Omega = \psi h_4 > 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{\Omega}}{\psi} \frac{A \sinh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right) + B \cosh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)}{A \cosh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right) + B \sinh\left(\frac{\sqrt{\Omega}}{h_1}\xi\right)} \tag{10}$$

Family 5: When $h_2 = 0$, $\psi = h_1 - h_3$ and $\Omega = \psi h_4 < 0$,

$$M(\xi) = \left(\frac{G'}{G}\right) = \frac{\sqrt{-\Omega}}{\psi} \frac{-A \sin\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right) + B \cos\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)}{A \cos\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right) + B \sin\left(\frac{\sqrt{-\Omega}}{h_1}\xi\right)} \tag{11}$$

3. Application of the method

In this section, we will implement the application of the new approach of generalized (G'/G) expansion method to construct