Physica C 479 (2012) 106–110

Contents lists available at SciVerse ScienceDirect

Physica C

journal homepage: www.elsevier.com/locate/physc

Vortex configuration and vortex–vortex interaction in nano-structured superconductors

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ARTICLE INFO

Article history:
Accepted 27 December 2011
Available online 16 January 2012

Keywords:
Vortex states
Nanostructures
Bogoliubov–de Gennes equation

ABSTRACT

We study the vortex structures and quasi-particle structures in nano-structured superconductors. We used the Bogoliubov–de Gennes equation and the finite element method and obtained stable magnetic flux structures and the quasi-particle states. We found the vortex configurations are affected by the interference of the quasi-particle bound states around the vortices. In order to clarify the interference between the quasi-particle wave-functions around two vortices we have developed a numerical method using the elliptic coordinates and the Mathieu functions. We apply this method to two singly quantized vortex state in a conventional *s*-wave superconductor and a pair of half-quantum vortices in a chiral *p*-wave superconductor.

1. Introduction

When the size of a superconductor is comparable to the penetration depth or the coherence length, the properties of the superconductors become different from those of the bulk superconductors. For example, it was pointed out that the transition temperature becomes higher for metallic cluster superconductors [1,2]. This is because the electrons are confined in the small regions and the energy spectrum and wave functions become different from those in the bulk metals. Also the vortex state becomes different when the size of the superconductor is comparable to the penetration depth [3–7]. The configuration of vortices is not the Abrikosov's triangular lattice and the giant vortex and anti-vortex appear.

For this problem, we consider how the vortex states are changed at the low temperature, where the order parameter fully developed and the quasi-particles appears at the core of vortices. Therefore we use the microscopic Bogoliubov–de Gennes (BdG) equation instead of the phenomenological Ginzburg–Landau equation. In order to solve the BdG equation in the arbitrary shaped small superconductors, we developed numerical method using the finite element method [8–10], we applied it to the nano-sized superconducting square plates and obtained the stable vortex structures and the quasi-particle structures [10]. Such BdG approach was also used in Ref. [11]. Especially we found that the quasi-particle bound states around the vortex interfere with those of other vortices. And we found that such interference affect the vortex configuration.

In order to clarify this interference the quasi-particle bound states around the multi vortices, we develop new numerical method for solving the BdG equation in the two vortices state. For such cases, the elliptic coordinates are useful. Two vortices are located at two foci of the ellipse. In the elliptic coordinates, the (modified) Mathieu functions are useful, and we use these functions for expansion of the wave functions. We apply this method for the two singly quantized vortices in a conventional *s*-wave superconductors and a pair of half-quantum vortices in a chiral *p*-wave superconductor [12–14].

In this paper, we first explain our numerical methods, and show how the quasi-particle bound states interfere with each other for *s*- and *p*-wave superconductors.

2. Method

We start the Bogoliubov–de Gennes equation

$$\left\{\frac{1}{2m}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}A\right)^2+V-\mu\right\}u+\Delta v=Eu, \tag{1}$$

$$-\left\{\frac{1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}A\right)^2+V-\mu\right\}v+\Delta^* u=Ev. \tag{2}$$

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doi:10.1016/j.physc.2011.12.023

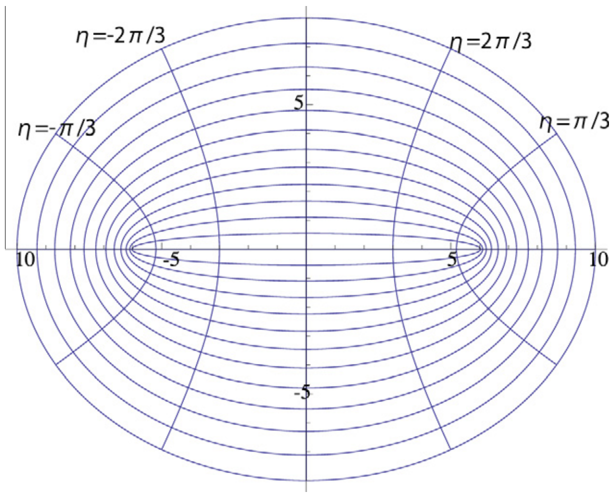

Fig. 1. Elliptic coordinates.

Here Δ is the superconducting order parameter and u and v are electron and hole components of quasi-particle wave functions, respectively.

For the finite element calculation, we expand the wave functions, the order parameter and the vector potential by the area coordinates.

$$\sum_j \left[P^1_{ij}(\{A\}) + P^2_{ij}(\{A\})\right] u^e_j + \sum_j Q_{ij}(\Delta)\, v^e_j = E \sum_j I_{ij} u^e_j, \tag{3}$$

$$\sum_j \left[-P^1_{ij}(\{A\}) + P^2_{ij}(\{A\})\right] v^e_j + \sum_j Q^*_{ij}(\Delta)\, u^e_j = E \sum_j I_{ij} v^e_j, \tag{4}$$

where $\{u^e_i\}$ and $\{v^e_i\}$ are values of wave functions at ith node of eth element. Details of formulation are available in Ref. [10].

For much more detailed calculation for two-vortices case, we use the elliptic coordinates, where two vortices are located at two foci $(\pm h_0, 0)$ of the ellipse (Fig. 1).

$$x = h_0 \cosh \xi \cos \eta, \tag{5}$$

$$y = h_0 \sinh \xi \sin \eta. \tag{6}$$

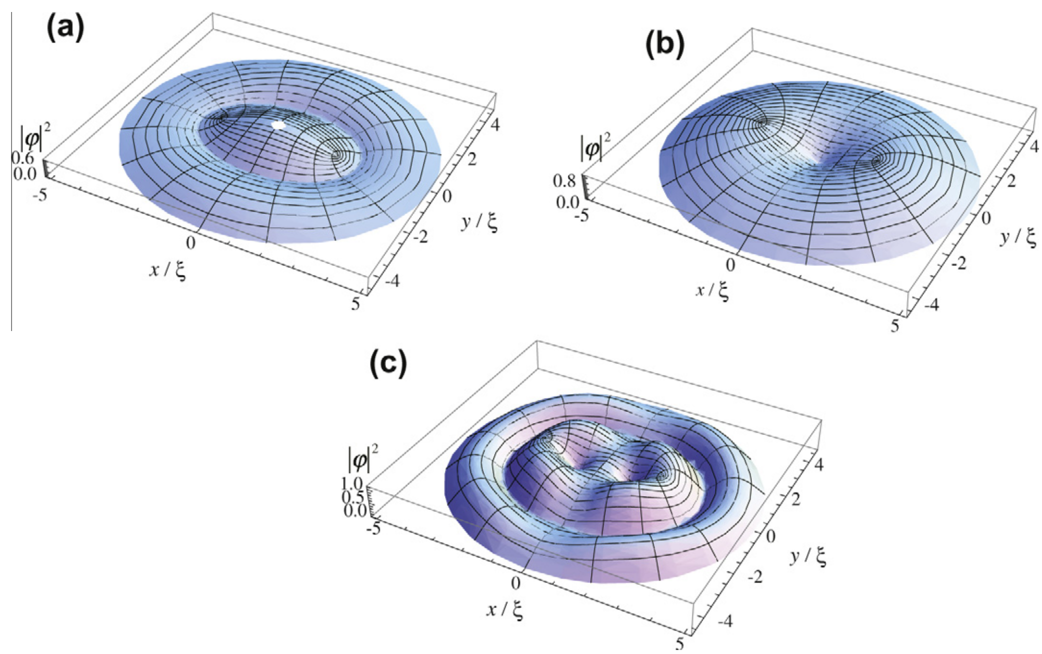

Fig. 2. Mathieu function basis $\mathrm{Ce}_m(\xi, q_{cmr})\mathrm{ce}_m(\eta, q_{cmr})$, (a) $m = 0$ and $r = 1$, (b) $m = 1$ and $r = 1$ and (C) $m = 2$ and $r = 2$.

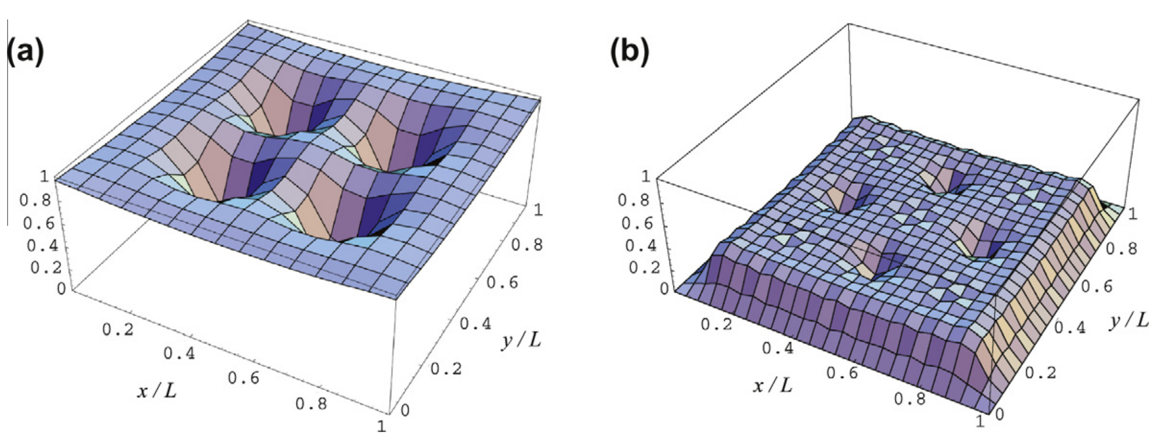

Fig. 3. Order parameter structure of four-vortices states, which are results of numerical calculations using the GL (a) and BdG equations.