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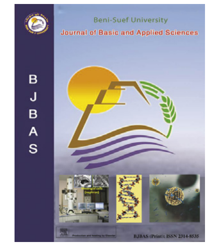

Full Length Article

Numerical solution of two-dimensional fuzzy Fredholm integral equations of the second kind using triangular functions

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ABSTRACT

The main purpose of this paper is to approximate the solution of linear two-dimensional fuzzy Fredholm integral equations of the second kind (2D-FFIE-2). We use fuzzy two-dimensional triangular functions (2D-TFs) to reduce the 2D–FFIE–2 to a system of linear Fredholm integral equations of the second kind with three variables in crisp. More over, we prove the convergence of the method. Finally we illustrate this method with some numerical examples to demonstrate the validity and applicability of the technique.

1. Introduction

Fuzzy integral equations are important in studying and solving a large proportion of the problems in many topics in applied mathematics, in particular in relation to physics, geographic, medical, biology. Usually in many applications some of the parameters in our problems are represented by fuzzy number rather than crisp, and hence it is important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy integral equations and solve them.

Recently, Mirzaee and Bimesl (2014) adapted the matrix method for the Fredholm integral equations. The study of fuzzy integral equations (FIEs) which attracted growing interest for some time, begins with the investigations of Kaleva (1987) and Seikkala (1987) for the fuzzy Volterra integral equation that is equivalent to the initial value problem for first order fuzzy differential equations. These studies continued by Wang (1984), Nanda (1989), Ralescu and Adams (1980), Bede and Gal (2005), Goetschel and Voxman (1986) and others. In Wu (2000) investigated the fuzzy Riemann integral and its numerical integration. Molabahrami et al. (2011) have used the parametric form of a fuzzy number and they have converted a linear fuzzy Fredholm integral equation to two linear systems of integral equations of the second kind in the crisp case.

Recently, some numerical methods have been investigated to solve linear fuzzy Fredholm integral equations of the

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second kind in two-dimensional space. For example, Rivaz and Yousefi (2012) and Ezzati and Ziari (2013) used homotopy perturbation method and fuzzy Bivariate Bernestein polynomials method for solving 2D-FFIE-2, respectively. Deb et al. (2006) introduced a new set of orthogonal functions, a numerical scheme based on such functions was applied for solving variational problem and integral equation by Babolian et al. (2007, 2009; 2010).

In this paper, we apply the triangular functions for approximate the solutions of the linear two-dimensional Fredholm fuzzy integral equations of the second kind for the first time.

This paper is organized as follows. In Section 2, we present some definitions and properties of one and two-dimensional triangular functions which will be used later. In Section 3, we give an overview of elementary concepts of the fuzzy calculus. two-dimensional fuzzy Fredholm integral equation is described in Section 4. In Section 5, we apply 2D-TFs for solving linear two-dimensional fuzzy Fredholm integral equation. Section 6 is concerned with discussing the convergency of the proposed method then this method is implemented for solving two illustrative examples in Section 7 and finally, conclusion is drawn in Section 8.

2. Preliminaries

2.1. A review of one-dimensional triangular functions

Definition 2.1. Two m-sets of triangular functions (*TFs*) are defined over the interval $[0,T]$ as:

$$T1_i(t) = \begin{cases} 1 - \dfrac{t - ih}{h} & ih \le t \le (i+1)h, \\ 0 & otherwise, \end{cases} \tag{1}$$

$$T2_i(t) = \begin{cases} \dfrac{t - ih}{h} & ih \le t \le (i+1)h, \\ 0 & otherwise, \end{cases} \tag{2}$$

where $i = 0,1,\ldots,m-1$, $h = T/m$, with a positive integer value for m. We have

$$\int_0^1 T1_i(t)T1_j(t)dt = \int_0^1 T2_i(t)T2_j(t)dt = \begin{cases} \dfrac{h}{3} & i = j, \\ 0 & i \ne j, \end{cases} \tag{3}$$

and

$$\int_0^1 T1_i(t)T2_j(t)dt = \int_0^1 T2_i(t)T1_j(t)dt = \begin{cases} \dfrac{h}{6} & i = j, \\ 0 & i \ne j. \end{cases} \tag{4}$$

Also, consider $T1_i$ as the i th left-handed triangular function and $T2_i$ as the i th right-handed triangular function. In this paper, it is assumed that $T = 1$.

Consider the first m terms of the left-handed triangular functions and the first m terms of the right-handed triangular functions and write them concisely as m-vectors:

$$T1(t) = \left[T1_0(t), T1_1(t), \ldots, T1_{m-1}(t)\right]^T, \tag{5}$$

$$T2(t) = \left[T2_0(t), T2_1(t), \ldots, T2_{m-1}(t)\right]^T, \tag{6}$$

where $T1(t)$ and $T2(t)$ are called left-handed triangular functions (*LHTF*) vector and right-handed triangular functions (*RHTF*) vector, respectively. We have:

$$\int_0^1 T1(t)T1^T(t)dt = \int_0^1 T2(t)T2^T(t)dt = \frac{h}{3}I, \tag{7}$$

$$\int_0^1 T1(t)T2^T(t)dt = \int_0^1 T2(t)T1^T(t)dt = \frac{h}{6}I, \tag{8}$$

which I is an $m \times m$ identity matrix. We denote the 1D-TF vector $T(t)$ as follows

$$T(t) = \begin{bmatrix} T1(t) \\ T2(t) \end{bmatrix}. \tag{9}$$

The expansion of the function $f(t)$ over $[0,1]$ with respect to 1D-TFs may be written as

$$f(t) \approx \sum_{i=0}^{m-1} c_i T1_i(t) + \sum_{i=0}^{m-1} d_i T2_i(t) = C^T \cdot T1(t) + D^T \cdot T2(t)$$
$$= \begin{bmatrix} C \\ D \end{bmatrix}^T \cdot \begin{bmatrix} T1(t) \\ T2(t) \end{bmatrix} = F^T \cdot T(t),$$

where C_i and D_i are samples of f, for example $C_i = f(ih)$ and $D_i = f((i+1)h)$ for $i = 0,1,\ldots,m-1$, so there is no need for integration. The vector F is called the 1D-TF coefficient vector.

2.2. Two-dimensional triangular functions and their properties

An $(m_1 \times m_2)$-set of the region ($\Omega = [0,1] \times [0,1]$) is defined by

$$T^{1,1}_{i,j}(s,t) = \begin{cases} (1 - \dfrac{s - ih_1}{h_1})(1 - \dfrac{t - jh_2}{h_2}) & ih_1 \le s \le (i+1)h_1, \\ & jh_2 \le t \le (j+1)h_2, \\ 0 & otherwise, \end{cases} \tag{10}$$

$$T^{1,2}_{i,j}(s,t) = \begin{cases} (1 - \dfrac{s - ih_1}{h_1})(\dfrac{t - jh_2}{h_2}) & ih_1 \le s \le (i+1)h_1, \\ & jh_2 \le t \le (j+1)h_2, \\ 0 & otherwise, \end{cases} \tag{11}$$

$$T^{2,1}_{i,j}(s,t) = \begin{cases} (\dfrac{s - ih_1}{h_1})(1 - \dfrac{t - jh_2}{h_2}) & ih_1 \le s \le (i+1)h_1, \\ & jh_2 \le t \le (j+1)h_2, \\ 0 & otherwise, \end{cases} \tag{12}$$

$$T^{2,2}_{i,j}(s,t) = \begin{cases} (\dfrac{s - ih_1}{h_1})(\dfrac{t - jh_2}{h_2}) & ih_1 \le s \le (i+1)h_1, \\ & jh_2 \le t \le (j+1)h_2, \\ 0 & otherwise, \end{cases} \tag{13}$$

where $i = 0,1,\ldots,m_1 - 1$, $j = 0,1,\ldots,m_2 - 1$ and $h_1 = 1/m_1$, $h_2 = 1/m_2$. m_1 and m_2 are arbitrary positive integers. It is clear that

$$\begin{aligned} T^{1,1}_{i,j}(s,t) &= T1_i(s).T1_j(t), \\ T^{1,2}_{i,j}(s,t) &= T1_i(s).T2_j(t), \\ T^{2,1}_{i,j}(s,t) &= T2_i(s).T1_j(t), \\ T^{2,2}_{i,j}(s,t) &= T2_i(s).T2_j(t). \end{aligned} \tag{14}$$

Furthermore,

$$T^{1,1}_{i,j}(s,t) + T^{1,2}_{i,j}(s,t) + T^{2,1}_{i,j}(s,t) + T^{2,2}_{i,j}(s,t) = \Phi_{i,j}(s,t),$$