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Two-dimensional Chebyshev hybrid functions and their applications to integral equations

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ABSTRACT

A combination of bivariate Chebyshev polynomials and two-dimensional block-pulse functions are introduced and applied for approximating the numerical solution of two-dimensional Fredholm integral equations. All calculations in this approach would be easily implemented. The method has the advantage of reducing computational burden. The convergence analysis is given. Some numerical examples are provided to illustrate the accuracy and computational efficiency of the proposed method.

1. Introduction

Two-dimensional Fredholm integral equations appear in mathematical modeling of various phenomena in physics and engineering. By using potential theorem, three dimensional Laplace's equation with boundary condition can be transformed into two dimensional boundary integral equation with weakly singular kernel function (Kress, 1989). The two dimensional images deblurring problem can be modeled as a two dimensional integral equation with smooth kernel function (Chan and Ng, 1996; Rajan and Chaudhuri, 2003). The transport equation is a linear case of the Boltzmann equation with wide applications in physics and engineering. This equation can be formulated as a two-dimensional Fredholm integral equation (Kadem and Baleanu, 2012). The two-dimensional Fredholm integral equation methods are developed and used for electromagnetic analysis, specifically for antennas and radar scattering (Volakis and Sertel, 2012). In linear case, these equations have the following form

$$f(s,t) = g(s,t) + \int_0^{T_2}\int_0^{T_1} k(s,t,x,y) f(x,y)\,dx\,dy, \quad (x,y)\in D, \qquad (1)$$

where $f(s,t)$ is an unknown scalar valued function defined on $D = [0,T_1] \times [0,T_2]$. The functions $k(s,t,x,y)$ and $g(s,t)$ are given functions defined on

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$W = \{(s,t,x,y): 0 \le x \le s \le T_1, 0 \le y \le t \le T_2\}$,

and D, respectively (Atkinson, 1997; Delves and Mohammed, 1985). Since any finite interval $[a,b]$ can be transformed to [0,1] by linear maps, it is supposed that $[0,T_1] = [0,T_2] = [0,1]$, without any loss of generality.

In comparison to one-dimensional integral equations, few numerical methods are known for approximating solution of Eq. (1). Hanson and Phillips (1978) proposed numerical solution of two-dimensional integral equations using linear elements. Guoqiang and Jiong (2001) used extrapolation for computing Nyström solution of two-dimensional nonlinear Fredholm integral equations. Gaussian radial basis functions was used for (1) by Alipanah and Esmaeili (2011). Some numerical methods based on piecewise polynomial interpolation was presented (Xie and Lin, 2009; Liang and Lin, 2010). Maleknejad and Mahdiani (2011) used two-dimensional block-pulse functions for solving mixed Volterra-Fredholm integral equations and nonlinear two-dimensional integral equations of the first kind (Maleknejad et al., 2010). Recently, the direct approaches for estimating numerical solution of two-dimensional Volterra-Fredholm and mixed integral equations were proposed using triangular orthogonal functions (Babolian et al., 2010; Maleknejad and JafariBehbahani, 2012).

In this paper, a new set of basis functions are constructed by combination of bivariate Chebyshev polynomials and two-dimensional block-pulse functions. A proposed name for these functions may be two-dimensional hybrid Chebyshev and block-pulse functions (2D-ChBPFs). Then solutions of two-dimensional linear Fredholm integral equations are computed approximately in a direct approach.

The proposed method, reduces a two-dimensional linear Fredholm integral equation to a system of algebraic equations. High accuracy of Chebyshev polynomials in expansion of functions leads to a good approximate solution for (1). Moreover, essential properties of block-pulse functions is due to setting up the algebraic system in a simple manner. The uniform convergence analysis and accuracy estimation of the method is presented. Finally, we check the proposed method on some examples to show its accuracy and superiority.

2. Preliminaries

The well-known Chebyshev polynomials of the first kind are defined by

$$T_n(t) = \cos(n\cos^{-1}t), \quad n \ge 0.$$

They have several applications in approximation theory and numerical analysis (Chihara, 1978). A set of Chebyshev polynomials is an orthogonal set with respect to the weight function $w(t) = \frac{1}{\sqrt{1-t^2}}$ on the interval $[-1,1]$, that is

$$<T_i(t), T_j(t)> = \int_{-1}^{1} \frac{T_i(t)T_j(t)}{\sqrt{1-t^2}}dt = \begin{cases} \pi, & i=j=0, \\ \frac{\pi}{2}, & i=j>0, \\ 0, & i \ne j. \end{cases}$$

They may be derived from the following recursive formula

$$T_0(t) = 1,$$
$$T_1(t) = t,$$
$$T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t), \quad m = 1,2,3,\cdots.$$

In a two-dimensional case, we can consider the bivariate Chebyshev polynomials on region $[-1,1] \times [-1,1]$ as

$$T_{i,j}(s,t) = T_i(s)T_j(t), \quad i,j \ge 0, \tag{2}$$

and any function $f(s,t)$ defined on $[-1,1] \times [-1,1]$ can be expanded with respect to bivariate Chebyshev polynomials as

$$f(s,t) = \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} a_{ij}T_{i,j}(s,t) \cong \sum_{j=0}^{M_2}\sum_{i=0}^{M_1} a_{ij}T_{i,j}(s,t),$$

for arbitrary positive integers M_1 and M_2 (Hadizadeh and Asgari, 2005). The coefficients a_{ij} may be computed as

$$a_{i,j} = \frac{<T_i(s), <f(s,t), T_j(t)>>}{<T_i(s), T_i(s)><T_j(t), T_j(t)>}, \quad i,j = 0,1,2,\cdots,$$

where $<\cdot,\cdot>$ denotes the inner product with respect to suitable weight function $w(t) = \frac{1}{\sqrt{1-t^2}}$ or $w(s) = \frac{1}{\sqrt{1-s^2}}$.

On the other hand, in an $(N_1 \times N_2)$-set of two dimensional block-pulse functions (2D-BPFs) on the region $[0,1) \times [0,1)$, each component is defined as

$$\phi_{n_1,n_2}(s,t) = \begin{cases} 1, & \frac{n_1-1}{N_1} \le s < \frac{n_1}{N_1}, \frac{n_2-1}{N_2} \le t < \frac{n_2}{N_2}, \\ 0, & otherwise, \end{cases}$$

where $n_1 = 1,2,\cdots,N_1$ and $n_2 = 1,2,\cdots,N_2$, for arbitrary positive integers N_1 and N_2 (Ganti, 1983). Some important properties of 2D-BPFs such as disjointness, orthogonality, and completeness are described in (Maleknejad et al., 2010; Ganti, 1983).

3. Hybrid Chebyshev polynomials and 2D-BPFs

In recent years, one-dimensional hybrid functions have been investigated in several studies as basis functions to estimate solutions of various equations.

Now, we can combine the bivariate Chebyshev polynomials and two-dimensional block-pulse functions to construct a new set of orthogonal functions over the region $[0,1) \times [0,1)$ as

$$b_{n_1,m_1;n_2,m_2}(s,t) = \begin{cases} T_{m_1,m_2}(2N_1s - 2n_1 + 1, 2N_2t - 2n_2 + 1), & \frac{n_1-1}{N_1} \le s < \frac{n_1}{N_1}, \ \frac{n_2-1}{N_2} \le t < \frac{n_2}{N_2}, \\ 0, & otherwise, \end{cases} \tag{3}$$

in which

N_1 denotes number of dissections of interval [0,1) over s axis,
M_1 denotes number of Chebyshev polynomials considered for variable s,
N_2 denotes number of dissections of interval [0,1) over t axis,