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Correcting a magnetic field map through the alignment of tracks

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ABSTRACT

A magnetic spectrometer is made of different track detectors at some nominal positions, immersed in a magnetic field. In addition to direct measurements, their actual positions are usually refined *a posteriori* through an alignment procedure exploiting the fact that tracks at different positions with different directions have different sensitivities to translations and rotations of the detectors. The sensitivity of the trajectories to variations of the field suggests the possibility to obtain an estimation of small discrepancies of the actual field map from the nominal one, exploiting the fact that tracks with different momenta undergo different deviations. Here a basis of field corrections is built with polynomials of increasing degree, satisfying the Maxwell equations, and the dependence of the trajectories on these corrections is linearized. Then the coefficients of the corrections are included as free parameters in an alignment procedure, in addition to the usual geometrical degrees of freedom. Due to the degeneracy between field scale and momentum scale, the corrected field map needs to be globally normalized, either by using reliable measurements at some positions, or by tuning the equivalent mass of pairs of particles from identified decays. A simple model of a forward magnetic spectrometer is built to evaluate in a realistic context the efficiency of the method in terms of systematic and statistical uncertainties. Possible generalizations are discussed.

1. Introduction

Most colliders or fixed target experiments include a magnetic spectrometer, made of several tracking devices inserted within or around a magnetic field, which provides a measurement of the particle momenta. A precise knowledge of the positions of the various subdetectors and of the field map is needed to achieve the best possible precision on the physical quantities of interest for charged particles, especially momenta and impact parameters. Hardware devices provide external measurements, but a software alignment procedure is usually operated to adjust the geometrical description, using samples of tracks of charged particles going through the spectrometer. In this study we examine the possibility to go further and to exploit the sensitivity of the trajectories to a distortion of the magnetic field with respect to the nominal map, by including additional degrees of freedom in the alignment parameters. The underlying idea is the following: the standard alignment procedures use samples of tracks with different positions and directions to give constraints on the geometry of the detector, considered as a set of rigid bodies; in addition, using tracks with different momenta gives constraints on the field, because the variations of curvature cannot be completely absorbed in rotations and translations of the subdetectors.

As an example we consider a simplified description of the spectrometer: an upstream and a downstream detector, each one measuring several points on the tracks, surrounding a magnetic region, as illustrated in Fig. 1. Both detectors are supposed to be internally aligned, that is, the only geometric degrees of freedom introduced in the algorithm describe their relative position. Such a model gives a realistic estimation of the precision that can be expected in real conditions, in combination with the more complete geometric alignment procedures which have been developed in real configurations, especially to cope with so called "weak modes" (for example using kinematical and/or vertex constraints as exploited in [1]).

In Section 2 we give a general description of the alignment procedure and the subsequent field normalization. In Section 3 we discuss the notion of *field corrections* and we present two ways of building a basis of polynomial functions of x, y, z satisfying the Maxwell equations, with definite parities in x, y, z. In Section 4 we apply the formalism to the simple model of forward spectrometer with the layout defined above. In that case, the geometric parameters are the relative position and orientation of the second one with respect to the first one (3 translations + 3 rotations), and the magnetic ones are the coefficients of the linear combination of corrections. In Section 6 we evaluate the systematic errors (due to the limited number of correcting terms) and the statistical ones (going as $1/\sqrt{N}$, N being the number of tracks used). We show that the normalization may be achieved using kinematical

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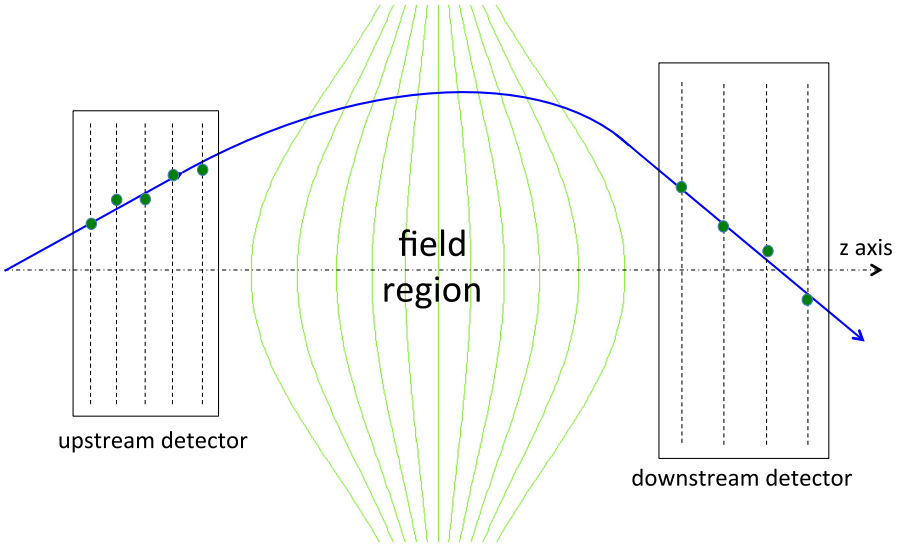

Fig. 1. Layout of the model of spectrometer. The field lines are produced by two magnetic dipoles (see Section 4); for convenience they are represented in yz projection, while the bended trajectories are drawn in xz projection.

constraints (here: the decay of neutral particles), and we discuss some possible residual biases. In Section 7 we sketch out possible extensions of the formalism to actual spectrometer layouts.

The aim of this paper is to point out the possibility to obtain corrections, not to optimize the procedure in a given real detector configuration. For example, the sensitivity of trajectories to field modifications is intrinsically more complex than the effect of geometrical displacements, and evaluating them requires much more computations. In this study, we use systematically the Runge–Kutta method at order 4, with a step length of 20 cm along the z axis, which takes about 20 microseconds per track following on a MacBook Pro (2.6 GHz Intel Core); some simplifications may be obtained, for example through parametrized extrapolations, but the optimization is specific to each experiment, and no general estimation of the computing load can be done. In any case, correcting the field map is not supposed to be repeated as often as making a geometrical alignment.

2. Principle of the procedure

Within a linear approximation around a reference trajectory (that may be achieved after iterations), the least squares fit of a trajectory provides a χ^2_{min} which is a quadratic function of the measurements. This remains true when using a Kalman Filter [2]. The principle of the alignment procedure is to introduce free parameters (corrections supposed to be small) which affect the measurements, and to adjust them in order to minimize the sum χ^2_{glob} of the χ^2_{min} of a large sample of tracks. If the effect of these corrections on the measurements may be linearized, χ^2_{glob} is a quadratic function of both the measurements and the correction parameters, and its minimization provides values through a set of linear equations. From this point of view, the magnetic corrections will be handled in the same way as the geometric ones, except that the dependence of measurements on such corrections is more complex.

However, it is clear that the field cannot be unambiguously defined by such a procedure, whatever the level of correction: a multiplication of the field components by a uniform factor is exactly compensated by the same factor applied to all momenta of the particles in the sample. As a consequence, although there is no degeneracy in the alignment fit, the result needs to be normalized. This may be done by using reliable and precise external measurement at some point(s), or kinematical constraints on the sample of tracks itself, for example by adjusting invariant masses of pairs of particles from identified decays.

3. Description of field corrections

3.1. General considerations on different options

Dealing with corrections refers usually to a *hierarchy* (successive orders, producing effects of decreasing order of magnitude). Intuitively it should refer to the amplitude of the modifications on position and direction for particles going through the field region, which are at first order integrals of the field components over the trajectories, divided by the signed momentum. From this point of view, if we assume that the deviation of the real field from the nominal one may be Taylor-expanded in powers of the coordinates, a natural choice is polynomials of x, y, z, ordered by degree. It was shown in [3] that triplets of polynomials for B_x, B_y, B_z can be selected to obey the Maxwell equations.

An advantage of polynomials is that the choice of the origin of coordinates is arbitrary, as far as one considers the space generated by *all* polynomials up to a given degree. If there is *a priori* a symmetry expressed as parities in x, y, z, or several coordinates, the space may be restricted to polynomials obeying this symmetry.

Polynomials suffer a drawback: they diverge for large values of the coordinates. But for our purpose, they do not need to be extrapolated outside the region actually covered by the tracks: our aim is to obtain effective corrections on the possible trajectories, not an accurate map over the whole field region.

In some configurations, the combinations proposed in [4] (products of trigonometric and hyperbolic functions) may be useful: for example $\cos(\alpha x)\cos(\beta y)\cosh(\gamma z)$ (or any function obtained by replacing some 'cos' by 'sin') is harmonic if $\alpha^2 + \beta^2 = \gamma^2$. Such functions diverge strongly at large $|z|$, so they should be applied into a well delimited domain in z.

With the two options described above, the correction has to be restricted to a well defined region, and boundary conditions cannot be imposed. As a consequence, the corrections introduce discontinuities on the edges of this region, and a stepwise computation of trajectories (e.g. the Runge–Kutta method) needs to set a step exactly on the boundary to avoid numerical errors that would spoil the intrinsic precision of the method.

On the contrary, one can try to use over the whole space functions that go to zero at large distance, as real magnetic fields should do. For example, any distortion can be considered as a superposition of fields generated by magnetic dipoles (or higher multipoles); these fields satisfy the Maxwell equations by construction. A hierarchical basis of corrections may consist of 1, 2, ..., dipoles, whose positions and