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Full Length Article

Vibration of a circular beam with variable cross sections using differential transformation method

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ABSTRACT

In this paper, an application of differential transformation method (DTM) is applied on free vibration analysis of Euler-Bernoulli beam. This beam has variable circular cross sections. Natural frequencies and corresponding mode shapes are obtained for three cases of cross section and boundary conditions. MATLAB program is used to solve the differential equation of the beam using DTM. Comparison of the obtained results with the previous solutions proves the accuracy and versatility of the presented problem.

1. Introduction

Beams with abrupt changes of cross-section are used widely in engineering. It can be easily made in order to save weight or to satisfy various engineering requirements. However, in many theses, beams are treated as volumes, so their cross section is usually rectangular. This paper presents the results of a case study of a beam element with circular cross section. Circular shape of the cross section is chosen because it often occurs in practice.

In this paper, the vibration problems of circular Euler-Bernoulli beams have been solved analytically using DTM. The beam has variable cross sections and various end conditions. In order to calculate the fundamental natural frequencies and the corresponding mode shapes, variational techniques were applied in the past such as Rayleigh Ritz, differential quadrature (DQM) and Galerkin methods. Also, some numerical methods were also successfully applied to beam vibration analysis such as finite element method (FEM).

The differential transformation method leads to an iterative procedure for obtaining an analytic series solutions of functional equations. Pukhov (1982) have developed a so-called differential transformation method (DTM) for electrical circuits' problems. In recent years, researchers Ayaz (2004), Moustafa (2008) and Qibo (2012) had applied the method to various linear and nonlinear problems. Comparison between (DTM) and (DQM) were applied by Attarnejad and Shahba (2008), and Rajasekaran (2009).

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Mahmoud et al. (2013) used the differential transformation method for the free vibration analysis of rectangular beams with uniform and non-uniform cross sections. Akiji et al. (1982) discussed the importance of the geometrical injection efficiency of a neutral circular beam. Un-damped vibration of beams with variable cross-section was analyzed by Datta and Sil (1996). Au et al. (1999) used C^1 modified beam vibration functions to study the vibration and stability of non-uniform beams with abrupt changes of cross-section. Li (2000) presented an exact approach for determining natural frequencies and mode shapes of non-uniform shear beams with arbitrary distribution of mass or stiffness. A three-dimensional method of analysis was presented for determining the free vibration frequencies and mode shapes of thick, tapered rods and beams with circular cross-section Kanga and Leissab (2004). Kisaa and Arif Gurelb (2006) presented a numerical model that combines the finite element and component mode synthesis methods for the modal analysis of beams with circular cross section and containing multiple non-propagating open cracks. Vibration and bending analysis were applied for non-uniform beams, rods and tubes by Mehmet et al. (2007), AL Kaisy et al. (2007), De Rosaa et al. (2008) and Shojaeifard et al. (2012). The beam elements, which are widely used in the absolute nodal coordinate formulation, were treated as iso-parametric elements by Grezegorz (2012).

In the present paper, an attempt is made to employ the differential transformation method to solve equations of motion for the free vibration of non-uniform circular beam. Three cases of boundary conditions are considered. Natural frequencies and corresponding mode shapes are obtained.

2. Basic equations

2.1. *Free vibration of non-uniform circular beam*

The governing differential equation for an Euler beam with a circular cross section with variable radius r_X as shown in Fig. 1 is given by:

$$\rho A(X)\frac{\partial^2 W(X,T)}{\partial T^2}+\frac{\partial^2}{\partial X^2}\left(EI(X)\frac{\partial^2 W(X,T)}{\partial X^2}\right)=0 \quad (1)$$

where ρ is the density of the beam material, $A(X)$ is the cross sectional area of the beam, $W(X,T)$ is displacement of the beam, E is young's modulus of the beam and $I(X)=\frac{\pi}{4}r_X{}^2$ is the inertia of the beam.

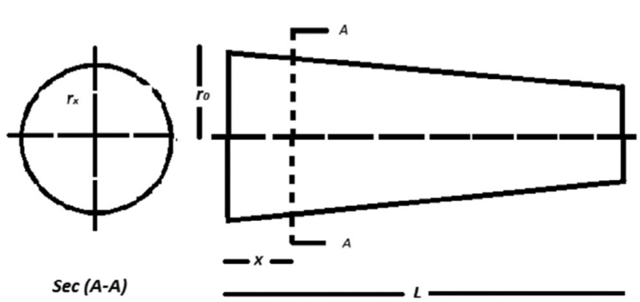

Fig. 1 – A sketch of beam with variable circular cross-section.

2.2. *Boundary conditions*

Case a. Simply Supported Beam

$$W(0,T)=0;\quad \frac{\partial^2 W(0,T)}{\partial X^2}=0;\quad W(L,T)=0;\quad \frac{\partial^2 W(L,T)}{\partial X^2}=0 \quad (2)$$

Case b. Clamped–Clamped Beam

$$W(0,T)=0;\quad \frac{\partial W(0,T)}{\partial X}=0;\quad W(L,T)=0;\quad \frac{\partial W(L,T)}{\partial X}=0 \quad (3)$$

Case c. Clamped–Roller Beam

$$W(0,T)=0;\quad \frac{\partial W(0,T)}{\partial X}=0;\quad W(L,T)=0;\quad \frac{\partial^2 W(L,T)}{\partial X^2}=0 \quad (4)$$

where L is the beam length and T is the time.

Table 1 – The first three non-dimensional frequencies (Ω_1, Ω_2 and Ω_3) of *Simply Supported uniform* Circular Euler beams for different number of terms.

No. of terms N	Ω_1	Ω_2	Ω_3
8	–	–	–
9	9.8902098156	28.2140699048	–
13	9.8696683979	37.2824413197	–
14	9.8696683979	37.2824413197	–
15	9.8696020437	40.0646967922	58.0531514621
17	9.8696044699	39.4169139196	–
18	9.8696044699	39.4169139196	–
23	9.8696044011	39.4784501712	87.8912222720
28	9.8696044011	39.4784176959	88.8107229014
33	9.8696044011	39.4784176044	88.8264496362
34	9.8696044011	39.4784176044	88.8264496362
38	9.8696044011	39.4784176044	88.8264396509
43	9.8696044011	39.4784176044	88.8264396098
44	9.8696044011	39.4784176044	88.8264396098
45	9.8696044011	39.4784176044	88.8264396098

Table 2 – The first three non-dimensional frequencies (Ω_1, Ω_2 and Ω_3) of *Clamped–Clamped uniform Circular* Euler beams for different number of terms.

No. of terms N	Ω_1	Ω_2	Ω_3
8	–	–	–
13	22.3776282691	53.7015697919	–
17	22.3733025493	60.9657850659	–
18	22.3733025493	60.9657850659	–
23	22.3732854786	61.6611232022	–
24	22.3732854414	61.6779003065	109.8144306817
28	22.3732854481	61.6728539182	120.0930730609
33	22.3732854481	61.6728228669	120.9036627032
36	22.3732854481	61.6728228682	120.9032822584
37	22.3732854481	61.6728228679	120.9033934841
38	22.3732854481	61.6728228679	120.9033934841
39	22.3732854481	61.6728228679	120.9033934841