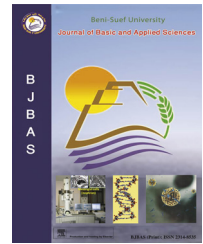


HOSTED BY

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

ScienceDirect

journal homepage: [www.elsevier.com/locate/bjbas](http://www.elsevier.com/locate/bjbas)

CrossMark

## Full Length Article

# Dynamic response of non-uniform beam subjected to moving load and resting on non-linear viscoelastic foundation

S.M. Abdelghany <sup>a,\*</sup>, K.M. Ewis <sup>a</sup>, A.A. Mahmoud <sup>b</sup>, M.M. Nassar <sup>b</sup><sup>a</sup> Engineering Mathematics and Physics Department, Faculty of Engineering, Fayoum University, Fayoum, Egypt<sup>b</sup> Engineering Mathematics and Physics Department, Faculty of Engineering, Cairo University, Giza, Egypt

## ARTICLE INFO

## Article history:

Received 22 November 2014

Accepted 16 January 2015

Available online 29 July 2015

## Keywords:

Dynamic response

Non-uniform beam

Viscoelastic foundation

## ABSTRACT

In this paper, it was purposed to comprehend the dynamic response of non-uniform Euler–Bernoulli simply supported beam which is subjected to moving load. The beam is rested on a nonlinear viscoelastic foundation. Galerkin with Runge–Kutta methods are employed. The influences of variations of the traveling velocity and the effect of increase in the magnitude of the moving load on the dynamic response are studied. A MATLAB code is designed to compute and plot the deflection.

© 2015 The Authors. Production and hosting by Elsevier B.V. on behalf of Beni-Suef University. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

## 1. Introduction

All branches of transport have experienced great advances characterized by increasingly higher speeds and weights of vehicles. As a result, structures and media over in which the vehicles move have been subjected to vibrations and dynamic stresses far larger than ever before. Many problems of considerable practical importance can be related to the solution of beams on a nonlinear viscoelastic foundation. Reinforced-concrete pavements of highways and airport runways and foundation slabs of buildings are well-known direct applications. That is, an analogy exists between the governing differential equation of a beam on a nonlinear viscoelastic foundation.

Transverse vibration analysis of uniform and non-uniform Euler–Bernoulli beams was explained by Safa et al. (2011). They explained the theory and analytical techniques about lateral

vibration of Euler–Bernoulli beams, and then described the methods used in the analysis. The dynamic effect of moving loads was not known until mid-nineteenth century. When the Stephenson's bridge across river Dee Chester in England in 1947 collapsed, it motivates the engineers for research of moving load problem. Moving loads have a great effect on the bodies or structures over which it travels. Fryba (1999) has presented fundamental studies in this area including most of the published articles before year 1999. The dynamic response of finite elastic structures (Rayleigh beams and plates) having arbitrary end supports and under an arbitrary number of moving masses was developed by Gbadeyan and Oni (1995). Serdar (2005) comprehend the dynamic response of beams and frames which are subjected to moving point load. Serdar employed finite element method and numerical time integration method (Newmark method) in the vibration analysis. The equation of motion in matrix form for an Euler beam subjected to a con-

\* Corresponding author. Tel.: +20 1140666453.

E-mail address: [Souma1400@yahoo.com](mailto:Souma1400@yahoo.com) (S.M. Abdelghany).

Peer review under the responsibility of Beni-Suef University.

<http://dx.doi.org/10.1016/j.bjbas.2015.05.007>

2314-8535/© 2015 The Authors. Production and hosting by Elsevier B.V. on behalf of Beni-Suef University. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

centrated mass moving at a steady speed was formulated by using the Green's function approach by [Sudhansu \(2012\)](#). He evaluated the solutions of the declared problems for the case of simply supported and cantilever beams by using dynamic Green's function.

It is clear that the main factor, which strongly affects the dynamic soil structure problem, is the validity of the foundation model. Free vibration of an Euler-Bernoulli beam resting on a variable Winkler foundation was studied by [Alev et al. \(2011\)](#). He solved the governing differential equations of the beam by using Differential Transform Method. The case of a nonlinear elastic foundation is of great theoretical and practical significance in railway engineering. In practice the foundation is highly nonlinear. [Pellicano and Mastroddi \(1997\)](#) studied nonlinear dynamics of beams resting on a nonlinear spring bed. [Coskun and Engin \(1999\)](#) presented the nonlinear vibration of a beam resting on nonlinear tensionless foundation subjected to a concentrated load at the center using the perturbation technique. [Tsiatas \(2010\)](#) presented the nonlinear problem of non-uniform beams. The nonlinear governing equations of beams rested on a nonlinear elastic foundation were solved using the Variational Iteration Method by [Younesian et al. \(2012\)](#). [Cao and Zhong \(2008\)](#) used the double Fourier transform and its inversion to solve the problem of dynamic response of an infinite beam placed on a Pasternak foundation when the system was subjected to a moving load. [Oni and Awodola \(2010\)](#) investigated the dynamic response under a concentrated moving mass of an elastically supported non-prismatic Bernoulli-Euler beam resting on an elastic foundation with stiffness variation. [Ansari et al. \(2010, 2011\)](#) studied vibration of a finite Euler-Bernoulli beam traversed by a moving load; the solution was obtained using the Galerkin method in conjunction with the Multiple Scales Method. [Yang et al. \(2010\)](#) presented the dynamical behavior of the vehicle-pavement-foundation coupled system using the Galerkin method and quick direct integral method. [Contreras et al. \(2010\)](#) described the experimental procedure followed for direct determination of dynamic modulus of asphalt mixtures by ultrasonic direct test at a specified temperature. [Ding et al. \(2012\)](#) introduce an investigation of the convergence of the Galerkin method for the dynamic response of a uniform beam resting on a nonlinear foundation with viscous damping subjected to a moving concentrated load.

In this paper, dynamic response of non-uniform Euler-Bernoulli simply supported beam will be obtained. The beam is subjected to moving load and rested on a nonlinear viscoelastic foundation. System parameters and magnitude of the moving load effects on the vertical deflections of non-uniform beam will be studied. [Mahmoud et al. \(2013\)](#) used the differential transformation method for analysis of the free vibration of beams with uniform and non-uniform cross sections.

## 2. Basic equations

Consider an Euler-Bernoulli beam with a non-uniform cross-section. This beam is resting on a nonlinear viscoelastic foundation and subjected to a moving load [Fig. 1](#). The foundation is taken as a nonlinear Winkler's foundation with linear-plus-cubic stiffness and viscous damping with three parameters as follows [Ding et al. \(2012\)](#):

$$P = \bar{k}_1 \bar{u}(\bar{x}, \bar{t}) + \bar{k}_3 \bar{u}^3(\bar{x}, \bar{t}) + \bar{\mu} \frac{\partial \bar{u}(\bar{x}, \bar{t})}{\partial \bar{t}} \quad (1)$$

where  $P$  is the foundation force induced per unit length of the beam,  $\bar{u}(\bar{x}, \bar{t})$  is displacement of the beam,  $\bar{k}_1$ ,  $\bar{k}_3$  and  $\bar{\mu}$  are the linear, nonlinear and damping parameters of the foundation, respectively,  $\bar{t}$  is the time.

According to Hamilton principle and the Euler-Bernoulli beam theory, the governing differential equation of motion for the non-uniform beam as

$$\rho A(\bar{x}) \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} + \frac{\partial^2}{\partial \bar{x}^2} \left( E I(\bar{x}) \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} \right) + \bar{k}_1 \bar{u} + \bar{k}_3 \bar{u}^3 + \bar{\mu} \frac{\partial \bar{u}}{\partial \bar{t}} = \bar{Q} \delta(\bar{x} - \bar{v} \bar{t}) \quad (2)$$

Where  $\rho$  is the density of the beam material,  $A(\bar{x})$  and  $I(\bar{x})$  are the cross sectional area and the inertia of the beam at a point at distance  $\bar{x}$ , respectively,  $E$  is young's modulus of the beam,  $\delta(\bar{x} - \bar{v} \bar{t})$  is the Dirac delta function used to deal with the moving concentrated load,  $\bar{Q}$  and  $\bar{v}$  are magnitude and speed of the load, respectively.

The boundary conditions of simply supported beam are:

$$\begin{aligned} \bar{u}(0, \bar{t}) = \frac{\partial^2 \bar{u}(0, \bar{t})}{\partial \bar{x}^2} &= 0 \\ \bar{u}(L, \bar{t}) = \frac{\partial^2 \bar{u}(L, \bar{t})}{\partial \bar{x}^2} &= 0 \end{aligned} \quad (3)$$

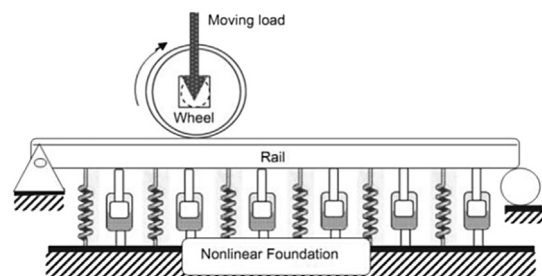
Introducing the dimensionless variables and parameters as follows:

$$\begin{aligned} u &= \frac{\bar{u}}{L}, \quad x = \frac{\bar{x}}{L}, \quad h(x) = h_0(1 - \beta x), \quad 0 \leq x \leq 1, \quad 0 \leq \beta \leq 1, \\ A(x) &= A_0(1 - \beta x), \quad I(x) = I_0(1 - \beta x)^3, \quad S(x) = (1 - \beta x)^3, \\ t &= \frac{\bar{t}}{L} \sqrt{\frac{E}{\rho}} \end{aligned}$$

Where  $x$  and  $L$  are the dimensionless spatial coordinate and the beam length, respectively,  $h_0$ ,  $A_0$  and  $I_0$  are the height, the cross sectional area and the inertia of the beam at its beginning point at distance  $x = 0$ , respectively, and  $t$  is the dimensionless time. Eq. (2) can be transformed into the dimensionless equation as

$$(S(x))^{1/3} \frac{\partial^2 u}{\partial t^2} + k_f^2 \frac{\partial^2}{\partial x^2} \left( S(x) \frac{\partial^2 u}{\partial x^2} \right) + k_1 u + k_3 u^3 + \mu \frac{\partial u}{\partial t} = Q_z \delta(x - vt) \quad (4)$$

where,



**Fig. 1 – Beam on nonlinear viscoelastic foundation subjected to a moving load. Ansari et al. (2011).**

Download English Version:

<https://daneshyari.com/en/article/816606>

Download Persian Version:

<https://daneshyari.com/article/816606>

[Daneshyari.com](https://daneshyari.com)