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The impact of incorporating shell-corrections to energy loss in silicon

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ABSTRACT

Modern silicon tracking detectors based on hybrid or fully integrated CMOS technology are continuing to push to thinner sensors. The ionization energy loss fluctuations in very thin silicon sensors significantly deviates from the Landau distribution. Therefore, we have developed a charge deposition setup that implements the Bichsel straggling function, which accounts for shell-effects. This enhanced simulation is important for comparing with testbeam or collision data with thin sensors as demonstrated by reproducing more realistically the degraded position resolution compared with naïve ionization models based on simple Landau-like fluctuation. Our implementation of the Bichsel model agrees well with the multipurpose photo absorption ionization (PAI) model in Geant4 and is significantly faster. The code is made publicly available as part of the Allpix software package in order to facilitate predictions for new detector designs and comparisons with testbeam data.

1. Introduction

The innermost layers of most modern collider tracking detectors are silicon pixels, using either hybrid modules or fully integrated CMOS technology. Requirements on the material budget and radiation hardness are pushing sensors to become thinner. Energy fluctuations in thick sensors is well-described by the Landau–Vavilov distribution [1,2]. However, when the sensor is sufficiently thin so that the number of collisions is small and the deposited energy still has the imprint of the shell structure of the silicon atom, the Landau–Vavilov distribution is not a good approximation. For thin sensors, the Bichsel straggling function is more complete and has been shown to reproduce measured energy losses [3].

State-of-the-art simulation of material interactions is provided by the Geant4 [4] toolkit for most high energy physics experiments, including testbeam simulations. The experiments at the LHC[1] most commonly use variations of the `EMstandard`[2] physics process list for Geant4 simulation [7]. This list does not include shell electron effects and due to its simplistic model, has a much faster execution time. The physical processes incorporated in `EMstandard` are an excellent model for thick sensors, but as they result in Landau–Vavilov-like distributions for the energy loss, they are not applicable for thin sensors. Geant4 does include a physics list with a more detailed energy loss model: the Photo Absorption Ionization (`PAI`) model [8]. The Geant4 implementation of the PAI model [9] is based on a corrected table of photo-absorption cross section coefficients. The simulated energy loss is in good agreement with the experiment data on energy loss for moderately thin sensors. However, the `PAI` model is not widely used by the major experiments because it is computationally expensive. Furthermore, like `EMstandard`, the `PAI` model is a generic approach that works for various elements while the Bichsel model has been refined based on extensive specific knowledge about silicon.

A dedicated implementation of the Bichsel straggling function has recently been implemented as part of the charge deposition model

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[1] While not used during particle propagation through the detector, the Bichsel model is used by the CMS experiment in a dedicated standalone simulation for a lookup table of charge sharing [5].

[2] The actual energy loss routine is the `Universal Function` which follows the Urbán model [6]. This two-state model uses Rutherford ($\propto 1/E^2$) cross-sections with a fudge-factor to match the most probable dE/dx. The width of the distribution is inflated for small thicknesses with an ad-hoc correction.

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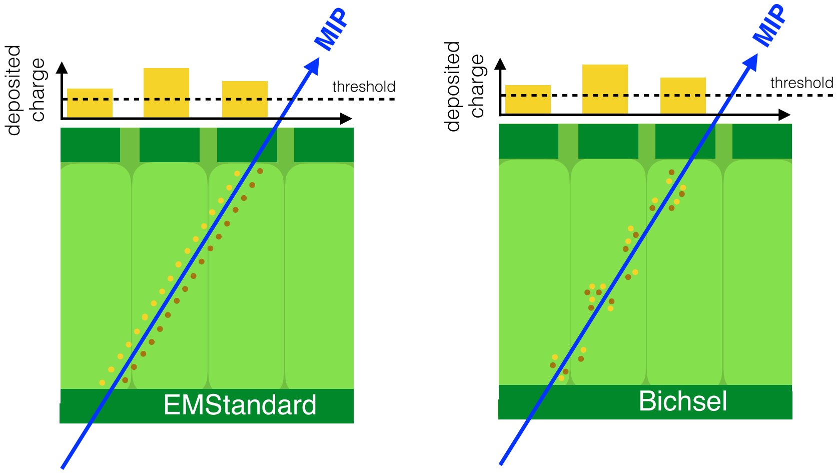

Fig. 1. Schematic diagrams of the energy deposition inside a pixel module using the `EMstandard` (left) and the Bichsel model (right).

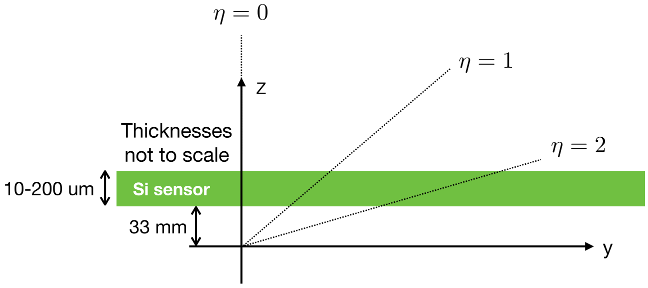

Fig. 2. The simulation setup used for the numerical studies. The detector thickness varies between 10 and 200 μm. A 25 GeV monochromatic muon source is placed at the origin of space. The muon travels in the $+zy$ plane The mean value of the particle pseudorapidity η is investigated at discrete values in $\{0, 1, 2\}$ and $\phi = \arctan(y/x) = 0$.

for the ATLAS detector [10]. In this model, the energy fluctuations significantly deviate from those introduced by `EMstandard`. The integrated cross section is used to compute both the location and amount of energy deposited. The stark contrast between the deposition pattern in `EMstandard` and in the Bichsel model is illustrated in Fig. 1. While the energy deposited at each point can vary in `EMstandard`, the distance between collisions is essentially fixed. The new straggling function has significant implications for position resolution and is a better model of the data.

In CMS, though the Bichsel model has not been used explicitly, their pixel simulation makes use of some smearing factors on Pixelav [11] which does include the Bichsel model. Hans Bichsel had also done various studies for STAR TPC [12] and other experiments have also studied the Bichsel model in the past [13]. However, there is no general-purpose community tool and there have not been any systematic studies of the impact of track resolution for various thicknesses.

This paper implements the Bichsel model into a standalone Geant4 package called Allpix [14] which is a common tool used for testbeam simulation. Section 2 reviews the model physics and provides a description of the technical implementation. The simulation framework and comparison metrics between our implementation, `EMstandard`, and `PAI` are described in Section 3 and the numerical results are shown in Section 4. This comparison includes an evaluation of the energy loss, position resolution, and CPU timing. The paper concludes in Section 5 with outlook for the future.

2. Bichsel model implementation

According to the convolution method in [3], fluctuations in the energy loss of the high energy particles traversing silicon are mainly due to two sources. The first source is the number of collisions the particle undergoes inside the material and the second source is the energy loss distribution per collision. The number n of inelastic scatters inside material over length x follows a Poisson distribution:

$$\Pr(n) = \frac{(x/\lambda)^n}{n!} e^{-x/\lambda}, \tag{1}$$

where λ is the mean free path, calculated from the collision cross section $\sigma(E)$ and the number of scattered centers per unit volume N as

$$\lambda^{-1} = N \int dE \sigma(E). \tag{2}$$

The spectrum for energy loss Δ after n collisions is calculated by the n-fold convolution of single collision spectrum $\sigma(E)$:

$$\sigma(\Delta)^{*n} = \int_0^{\Delta} \sigma(E) \sigma^{*(n-1)}(\Delta - E) dE, \tag{3}$$

with $\sigma(\Delta)^{*0} = \delta(\Delta)$ (Dirac δ-function) so that $\sigma(\Delta)^{*1} = \sigma(\Delta)$. Therefore, the full straggling function is:

$$f(\Delta, x) = \sum_{n=0}^{\infty} \frac{(x/\lambda)^n e^{-x/\lambda}}{n!} \sigma(\Delta)^{*n}. \tag{4}$$

Eq. (4) does not admit a closed-form analytic solution, but numerical calculations are provided in [3]. Our implementation follows that of [10] using a Monte Carlo simulation of the individual scatters inside the sensor. For an incident particle with velocity $\beta\gamma$, a path length x is sampled according to an exponential distribution whose average is the mean free path. If moving along the particle trajectory by this amount is still inside the sensor, an energy E is sampled from the probability distribution $\sigma(E)/\int_0^{E_{\max}(\beta\gamma)} dE' \sigma(E')$. If this sampled energy is less than the incident particle energy, then the particle is advanced by x and an energy E is recorded. This process is then repeated until the particle is no longer in the sensor. Knock-out electrons (δ-rays) are provided by Geant4 (using `EMstandard`) and occur $\mathcal{O}(1\%)$ of the time per 100 μm of path length in silicon. To avoid double-counting, the energy loss spectrum in the Bichsel model is cut-off at the Geant4 δ-ray production threshold (chosen to be 117 keV). Also in the Bichsel model, it is possible that no energy is deposited if the sensor is sufficiently thin. The probability for depositing any energy in a 1 μm thick sensor is 97.8% and the probability that this energy is above 10 eV is about 86.9%.