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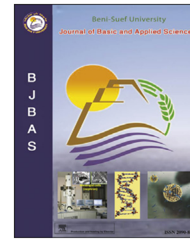

# A Fibonacci collocation method for solving a class of Fredholm–Volterra integral equations in two-dimensional spaces

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ABSTRACT

In this paper, we present a numerical method for solving two-dimensional Fredholm–Volterra integral equations (F-VIE). The method reduces the solution of these integral equations to the solution of a linear system of algebraic equations. The existence and uniqueness of the solution and error analysis of proposed method are discussed. The method is computationally very simple and attractive. Finally, numerical examples illustrate the efficiency and accuracy of the method.




## 1. Introduction

A sequence of polynomials is a Fibonacci sequence if it satisfies the recursion

$$f_{n+2}(x) = x \cdot f_{n+1}(x) + f_n(x), \quad n \geq 1. \tag{1}$$

Two well-known Fibonacci sequences are the Fibonacci polynomials, $\{F_n(x)\}$, defined using (1) with $F_1(x) = 1$ and $F_2(x) = x$ and the Lucas polynomials, $\{L_n(x)\}$, defined using (1) with $L_1(x) = 2$ and $L_2(x) = x$ (Bergum and Hoggatt, 1974; Bicknell, 1970; Bicknell and Hoggatt, 1973). In addition to being Fibonacci sequences, these polynomials produce Fibonacci and Lucas numbers, respectively, when evaluated at $x = 1$. Fibonacci (from "filius Bonacci"), Italian mathematician of the 13th century, is the best known to the modern world for a number sequence named the Fibonacci numbers after him, which he did not discover but used as an example in his book, Liber Abaci. In Fibonacci's Liber Abaci book, chapter 12, he posed, and solved a problem involving the growth of a population of rabbits based on idealized assumptions. The solution, generation by generation, was a sequence of numbers later known as Fibonacci numbers. The number sequence was known to Indian mathematicians as early as the 6th century, but it was Fibonacci's Liber Abaci that introduced it to the West. In the Fibonacci sequence of numbers, each number is

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the sum of the previous two numbers, starting with 0 and 1. This sequence begins 0, 1, 1, 2, 3, 5, ... (Grimm, 1973).

Recently, Mirzaee and Hoseini (2013a, 2013b; 2014) adapted the matrix method for the Fibonacci polynomials. They have been used the Fibonacci matrix method to find approximate solutions of singularly perturbed differential-difference equations and systems of linear Fredholm integro–differential equations by setting the equations for the Fibonacci polynomials in matrix form as $F(x) = BX(x)$, where $F(x) = [F_1(x), F_2(x), F_3(x), \ldots, F_{N+1}(x)]^T, X(x) = [1, x, x^2, x^3, \ldots, x^N]^T$, and $B$ is the invertible lower triangular matrix with entrances the coefficients appearing in the expansion of the Fibonacci polynomials in increasing powers of $x$. They approximate the solution of these equations as follows:

$$y_i(x) \simeq \sum_{n=1}^{N+1} a_{i,n} F_n(x), \quad i = 1, 2, \ldots, l, \quad 0 \le a \le x \le b, \tag{2}$$

where $a_{i,n}$, $n = 1, 2, \cdots, N+1$ are the unknown Fibonacci coefficients, N is any arbitrary positive integer such that $N \ge m$, and $F_n(x)$, $n = 1, 2, \ldots, N+1$ are the Fibonacci polynomials defined by

$$F_{n+1}(x) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-i}{i} x^{n-2i}, \quad n \ge 0, \tag{3}$$

where $\lfloor n/2 \rfloor$ denotes the greatest integer in $n/2$.

Note that $F_{2n}(0) = 0$ and $x = 0$ is the only real root, while $F_{2n+1}(0) = 1$ with no real roots. Also for $x = k \in N$ we obtain the elements of the $k$-Fibonacci sequences (Falcon and Plaza, 2009).

The Volterra–Fredholm integral equations arise in a variety of applications in many fields including continuum mechanics, potential theory, geophysics, electricity and magnetism, antenna synthesis problem, communication theory, mathematical economics, population genetics, radiation, the particle transport problems of astrophysics and reactor theory, fluid mechanics (Abdou, 2003; Bloom, 1980; Jaswon and Symm, 1977; Jiang and Rokhlin, 2004; Schiavane et al. (2002); Semetanian, 2007; Voitovich and Reshnyak, 1999).

In recent years, many different basic functions have used to estimate the solution of linear and nonlinear Volterra–-Fredholm integral equations, such as orthonormal bases and wavelets (Brunner, 1990; Ghasemi et al. (2007); Maleknejad et al. (2010); Ordokhani, 2006; Ordokhani and Razzaghi, 2008; Yalcinbas, 2002; Yousefi and Razzaghi, 2005). Mirzaee and Hoseini (2013a, 2013b) applied hybrid of block-pulse functions and Taylor series to approximate the solution of a nonlinear Fredholm–Volterra integral equation of the form

$$y(t) = x(t) + \lambda_1 \int_0^t k_1(t,s) F_1(y(s)) ds + \lambda_2 \int_0^1 k_2(t,s) F_2(y(s)) ds,$$

$$t \in I = [0,1],$$

where $\lambda_1$ and $\lambda_2$ are constant, $x(t) \in L^2(I), k_1(t,s)$ and $k_2(t,s) \in L^2(I \times I)$ and $F_1(y(s))$ and $F_2(y(s))$ are given continuous functions which are nonlinear with respect to $y(t)$, and $y(t)$ is an unknown function.

In this work, we consider the two dimensional Fredholm–Volterra integral equation that given by the form

$$g(s,t) + \int_0^1 q(s,y) g(y,t) dy + \int_0^t k(t,x) g(s,x) dx = f(s,t), \quad s,t \in I, \tag{4}$$

where $q$ , $k$, $f$ are known and g is unknown. Moreover, functions $q$, $k$, $f$ and $g$ belong to $L^2(\Gamma)$ that $\Gamma = I \times I$. Hendi and Bakodah (2013) have been used the Adomian decomposition method to find approximate solution of nonlinear Fredholm–Volterra integral equations. Babolian et al. (2011) applied block-pulse functions to solve Eq. (4). In this manuscript, we propose a method based on series of Fibonacci polynomials to solve the Fredholm–Volterra integral equation (4).

## 2. Method of solution

The aim of our method is to get solution as Fibonacci series defined by

$$g(s,t) \simeq g_{N+1,M+1}(s,t) = \sum_{n=1}^{N+1} \sum_{m=1}^{M+1} c_{nm} F_n(s) F_m(t) = F(s,t) C, \tag{5}$$

where $c_{nm}$, $n = 1, 2, \ldots, N+1$, $m = 1, 2, \ldots, M+1$ are the unknown Fibonacci coefficients,

$$C = \left[c_{11}, \ldots, c_{1(M+1)}, c_{21}, \ldots, c_{2(M+1)}, \ldots, c_{(N+1)1}, \ldots, c_{(N+1)(M+1)}\right]^T,$$

$N$ is any arbitrary positive integer, $F_n(x), n = 1, 2, \ldots, N+1$ are the Fibonacci polynomials defined in Eq. (3) and $F(s,t)$ is $1 \times (N+1)(M+1)$ matrix introduced as follows

$$F(s,t) = \left[h_{11}(s,t), \ldots, h_{1(M+1)}(s,t), h_{21}(s,t), \ldots, h_{2(M+1)}(s,t), \ldots, h_{(N+1)1}(s,t), \ldots, h_{(N+1)(M+1)}(s,t)\right],$$

where

$$h_{nm}(s,t) = F_n(s) F_m(t), \quad n = 1, 2, \ldots, N+1, m = 1, 2, \ldots, M+1.$$

The method of collocation solves the F-VIE (4) using the approximation (5) through the equations

$$r_{N+1,M+1}(s_i, t_j) = g_{N+1,M+1}(s_i, t_j) + \int_0^1 q(s_i, y) g_{N+1,M+1}(y, t_j) dy + \int_0^{t_j} k(t_j, x) g_{N+1,M+1}(s_i, x) dx - f(s_i, t_j). \tag{6}$$

that for a suitable set of collocation points, we choose Newton-Cotes nodes as $(s_i, t_j) = (2i - 1/2(N+1), 2j - 1/2(M+1))$ for all $i = 1, 2, \ldots, N+1$, $j = 1, 2, \ldots, M+1$.



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