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Optimum resonance control knobs for sextupoles

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ABSTRACT

We discuss the placement of extra sextupoles in a magnet lattice that allows to correct third-order geometric resonances, driven by the chromaticity-compensating sextupoles, in a way that requires the least excitation of the correction sextupoles. We consider a simplified case, without momentum-dependent effects or other imperfections, where suitably chosen phase advances between the correction sextupoles leads to orthogonal knobs with equal treatment of the different resonance driving terms.

1. Introduction

Nonlinear magnetic fields limit the performance of many storage rings by reducing their dynamic aperture. Beam particles passing this boundary of stability are doomed to hit the beam pipe or experiment in an uncontrolled way. In high-energy colliders the enormous energy stored in the beams requires to maintain these losses at a low level. In synchrotron light sources lost particles interfere with the often delicate experiments and in some cases require more frequent injections. In all cases strategies are required to compensate the magnetic nonlinearities due to fringe fields and eddy-currents in the superconducting high-energy rings or the sextupoles needed to correct the very large chromaticities in the strongly-focusing synchrotron light sources.

The mechanism with which nonlinearities force particles to ever increasing amplitudes is linked to the resonances they excite. Here by resonance we mean a force that coherently accumulates, dependent on the tunes Q_x, Q_y and Q_z of the storage ring. In general there are resonances in all three spatial dimensions but we restrict our discussion to only treat horizontal and vertical direction and the resonances are labeled by two integers m and n by $mQ_x \pm nQ_y$. Each multipole in the ring drives a number of resonances and it is possible to analytically calculate its contribution. This opens up the possibility to theoretically analyze different configurations and design compensation schemes in an optimum way.

Sometimes it is possible to build compensating schemes into the magnetic lattice during the design phase. An ingenious example is described in [1,2] where the chromaticity-correcting sextupoles are arranged in such a way that they compensate all geometric aberrations up to second order. If solving the problem before building the accelerator is not possible, schemes are needed to identify and determine the excitation of correction magnets. Linear combinations of driving these magnets in 'teams' are often called 'knobs'. A recent example is given in [3] where linear combinations of sextupoles are constructed to correct individual resonances. In that report the placement of the sextupoles is given before-hand and the authors construct orthogonal 'resonance-control knobs' from an elaborate mathematical analysis. Even the correction of the betatron-coupling in the LHC requires '[knobs] … *as orthogonal as possible using the minimum possible skew quadrupolar strength*' [4].

In our analysis we investigate whether there is an optimum placement of correction magnets, in our case sextupoles, that allows us to correct the resonances with the least effort in the sense that the required excitation of the correction magnets is as small as possible. This is advantageous because weaker correction elements will generate weaker higher-order aberrations. A trivial counter-example of a system that requires stronger magnet excitations than necessary is based on two steering magnets that are close to each other. Both can change the angle of the beam, but in order to cause a transverse position offset, both magnets need to be powered with large currents, but with opposite polarity. In essence the magnets are fighting each other. A simple cure is to place the magnets at locations with a betatron phase-advance of 90 degrees apart. In that case both changes, in angle and position, are equally possible.

In this report we discuss a scheme that generalizes the concept with perfect phase advances between sextupoles and find a configuration to control the amplitude and the phase of the $Q_x, 3Q_x, Q_x+2Q_y$, and Q_x-2Q_y resonances is accomplished by orthogonal knobs driving eight independently powered sextupoles. The orthogonalization is built into the lattice via a suitable placement of the sextupoles. The inspiration for this idea came from earlier work, by one of the authors, on the correction of skew quadrupole resonances in the LHC [5].

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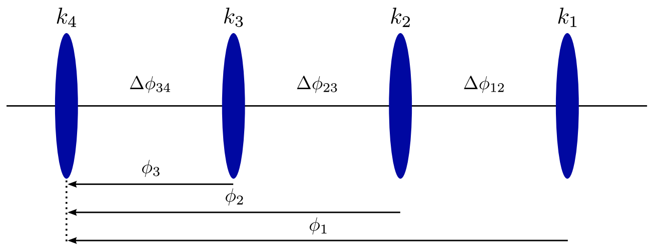

Fig. 1. A beamline with four sextupoles placed at locations with phase advances ϕ_1, ϕ_2 and ϕ_3 to the reference point which is chosen to be the location of the fourth sextupole.

In the following sections we first describe the method in the one-dimensional case where we correct amplitude and phase of the Q_x and $3Q_x$ resonance. In the subsequent section we generalize this to the four-dimensional case and verify the results by tracking simulations. Finally we offer a suggestion of how to implement this in a lattice.

2. One-dimensional sextupoles

We use a Hamiltonian formalism and tools from Lie algebra in order to analyze different sextupole setups. In particular we make use of the similarity transformation [6–8] that allows Hamiltonians to be moved to different locations and the Campbell–Baker–Hausdorff (CBH) formula to concatenate Hamiltonians into a single, effective Hamiltonian.

Let us begin by finding the resonance driving terms (RDTs) from a one-dimensional sextupole. The Hamiltonian for a sextupole is given by

$$H = k\beta^{3/2}\bar{x}^3 \tag{1}$$

where $k = \frac{1}{6}k_2 l$ is the integrated sextupole strength normalized to the beam energy, β is the beta function from the linear motion and x is the position in normalized phase space coordinates. We can move the Hamiltonian using the similarity transformation and in normalized phase space the linear map is a rotation with phase advance ϕ. At the new location we have $\bar{x} = x\cos\phi - x'\sin\phi$ and the Hamiltonian is given by

$$H = k\beta^{3/2}(x\cos\phi - x'\sin\phi)^3. \tag{2}$$

We expand this expression and make use of trigonometric identities such as $\cos^3(\phi) = \frac{1}{4}[\cos(3\phi) + 3\cos(\phi)]$. Furthermore, we express the coordinates in action–angle variables ($x = \sqrt{2J}\cos\psi, x' = -\sqrt{2J}\sin\psi$) and we find

$$H = \frac{k}{4}\left[\cos(3\phi)(2J\beta)^{3/2}\cos(3\psi) + \sin(3\phi)(2J\beta)^{3/2}\sin(3\psi) + 3\cos(\phi)(2J\beta)^{3/2}\cos(\psi) + 3\sin(\phi)(2J\beta)^{3/2}\sin(\psi)\right] \tag{3}$$

where we have four RDTs with amplitudes depending on the phase advance ϕ. We can add more sextupoles to the beamline and move them all to the same reference point. To first order the concatenation of the Hamiltonians involves only addition of the Hamiltonians and in order to calculate the third-order part of the effective Hamiltonian we do not need higher-order terms in the CBH formula.

To control all four RDTs independently we need at least 4 sextupoles. We assume four sextupoles with strengths $\{k_1, k_2, k_3 k_4\}$, placed at locations with equal beta functions and phase advances $\{\phi_1, \phi_2, \phi_3, \phi_4\}$ to the reference point which we, without loss of generality, set to be at the location of the fourth sextupole (i.e. $\phi_4 = 0$). Fig. 1 shows a schematic of the setup and the phase advances depend on the relative phase advances between the consecutive sextupoles as

$$\begin{aligned}\phi_1 &= \Delta\phi_{12} + \Delta\phi_{23} + \Delta\phi_{34}\\ \phi_2 &= \Delta\phi_{23} + \Delta\phi_{34}\\ \phi_3 &= \Delta\phi_{34}.\end{aligned} \tag{4}$$

Table 1
Phase advances.

Sextupole	ϕ [degr.]
1	135°
2	90°
3	45°
4	0°

The first resonance driving term in (3) is given by addition of the contributions from the four sextupoles and we find

$$H_{\cos(3\psi)} = \frac{1}{4}(2J\beta)^{3/2}\cos(3\psi) \times \left[k_1\cos(3\phi_1) + k_2\cos(3\phi_2) + k_3\cos(3\phi_3) + k_4\right] \tag{5}$$

and similarly for the other RDTs. We can express the effective Hamiltonian at the reference point to first order in sextupole strengths as linear system $\vec{C} = M\vec{k}$ where $\vec{C}$ contains the coefficients for the different third-order RDTs, M is a matrix with the trigonometric identities depending on the phase advances from each sextupole to the reference point and $\vec{k}$ contains the sextupole strengths. Explicitly we have

$$\begin{bmatrix} C\{\frac{1}{4}(2J\beta)^{3/2}\cos(3\psi)\} \\ C\{\frac{1}{4}(2J\beta)^{3/2}\sin(3\psi)\} \\ C\{\frac{3}{4}(2J\beta)^{3/2}\cos(\psi)\} \\ C\{\frac{3}{4}(2J\beta)^{3/2}\sin(\psi)\} \end{bmatrix} = \begin{bmatrix} \cos(3\phi_1) & \cos(3\phi_2) & \cos(3\phi_3) & 1 \\ \sin(3\phi_1) & \sin(3\phi_2) & \sin(3\phi_3) & 0 \\ \cos(\phi_1) & \cos(\phi_2) & \cos(\phi_3) & 1 \\ \sin(\phi_1) & \sin(\phi_2) & \sin(\phi_3) & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} \tag{6}$$

where $C\{\frac{1}{4}(2J\beta)^{3/2}\cos(3\psi)\}$ denotes the coefficient of the $\frac{1}{4}(2J\beta)^{3/2}$ $\cos(3\psi)$ term.

It is well-known that for $\Delta\phi_{12} = \Delta\phi_{23} = \Delta\phi_{34} = 180°$ all resonances cancel if all sextupoles have the same excitations $k_1 = k_2 = k_3 = k_4$ [9]. However, such phase advances are not suitable to control the resonance driving terms since in this case the matrix M is singular and implies that the system cannot be inverted and the individual RDTs cannot be controlled. Instead we look for a setup with four sextupoles with optimum orthogonality for the control of the different RDTs which means that we require the rows of matrix $M = M(\phi_1, \phi_2, \phi_3)$ to be orthogonal. Table 1 shows the phase advances between the four sextupoles and the reference point that yields a solution with a 45° separation between all sextupoles and this results in optimum orthogonality since the resulting matrix

$$M = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \tag{7}$$

has rows that are orthogonal. In fact, this matrix has condition number equal to unity, which indicates that all eigenvalues of the matrix are equal as shown in Appendix A. There we show that optimality requires the response matrix M to have condition number unity. This in turn guarantees that RDTs of equal magnitude require the same rms strength of correction. In other words, all RDTs are controlled equally well. Since the condition number is unity, the columns of M are also orthogonal. If we factor out $\sqrt{2}$ from M and absorb this in $\vec{C}$ instead we are left with an orthogonal matrix, that is, a matrix that fulfills $M^T M = I$.

In this section we found a setup with four sextupoles separated with phase advances yielding optimum orthogonality of the knobs for the different third order resonances. Next we investigate similar setups for two-dimensional sextupoles.