Nuclear Inst. and Methods in Physics Research, A 892 (2018) 41–47

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research, A

journal homepage: www.elsevier.com/locate/nima

Magnetic field of longitudinal gradient bend

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ARTICLE INFO

Keywords:
Electron storage ring
Low emittance lattice
Longitudinal gradient bend

ABSTRACT

The longitudinal gradient bend is an effective method for reducing the natural emittance in light sources. It is, however, not a common element. We have analyzed its magnetic field and derived a set of formulae. Based on the derivation, we discuss how to model the longitudinal gradient bend in accelerator codes that are used for designing electron storage rings. Strengths of multipole components can also be evaluated from the formulae, and we investigate the impact of higher order multipole components in a very low emittance lattice.

1. Introduction

New or upgraded third generation light sources will realize small electron beam emittance of pico-meter regime, delivering high brightness photon beams for the experiments. Multi-bend achromat (MBA) lattices, where multiple dipole bending magnets per arc are installed, are generally utilized, since the beam emittance is inversely proportional to the third power of the deflection angle per dipole magnet. As the name suggests, the dispersion function is suppressed at both ends so as not to enlarge the electron beam size due to the energy spread at the location of insertion devices.

The optical functions over the dipole magnet are adjusted to lower the emittance. The $\mathcal{H}$-function is taken as a figure of merit [1]:

$$\mathcal{H} = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2, \tag{1}$$

where β, α and γ are Twiss parameters, and η and η' are the dispersion function and its derivative. The amount of emittance generated by photon emission in the bending magnets is proportional to $\mathcal{H}$.

The optimal optical parameters to minimize the $\mathcal{H}$ function, resulting in the *theoretical minimum emittance*, can be analytically found at least for homogeneous dipole fields [2]. The minimization of $\mathcal{H}$ requires rather small beta and dispersion functions through the dipole magnets, and these values are determined by the length of the dipole magnet. However, they are not realized in practice because the required focusing is too strong and/or the arc length too long, and thus beam emittance is normally well above the theoretical minimum emittance.

The longitudinal gradient bend (LGB), in which the magnetic field varies along the beam orbit, is an effective method for reducing $\mathcal{H}$ and the resulting beam emittance. Several studies can be found in the literature, e.g. [3–6]. Intuitively, the emittance is lowered when more bending is applied at the location of low dispersion since the emission of a synchrotron radiation photon increases the betatron oscillation amplitude, depending on the magnitude of the energy loss and the dispersion function. Therefore, the optimum field profile has a peak in the middle of the dipole magnet [7].

Accelerator codes such as MADX [8], Bmad [9], Tracy [10] and Elegant [11], are widely used to design storage ring lattices. An LGB, however, is not yet a common accelerator element, and thus it is not available in these codes. We have analyzed an LGB's magnetic field and derived a set of formulae to model it properly. In this paper, we report on the analysis and discuss the LGB's multipole components, namely sextupole and octupole. In [6], they are only qualitatively discussed whereas we evaluate the impact of these higher order terms quantitatively.

2. Magnetic field description

2.1. Coordinate system and magnetic field

The coordinate system shown in Fig. 1 is used throughout this paper. The coordinate of the magnet is represented by fixed Cartesian coordinates, X–Y–Z. The plane X–Z corresponds to the dipole symmetry plane where the horizontal magnetic field components are zero, $B_X = B_Z = 0$. On this plane, the vertical field component is given by

$$B_Y = B_Y(X, Y = 0, Z). \tag{2}$$

The symbol B_Y will be used as the vertical field on the symmetry plane, and $Y = 0$ is omitted hereafter.

The design closed orbit of the beam is normally on the symmetry plane. We employ another coordinate system, x–y–s, moving along the

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https://doi.org/10.1016/j.nima.2018.02.111
Received 1 December 2017; Received in revised form 12 February 2018; Accepted 27 February 2018
Available online 8 March 2018

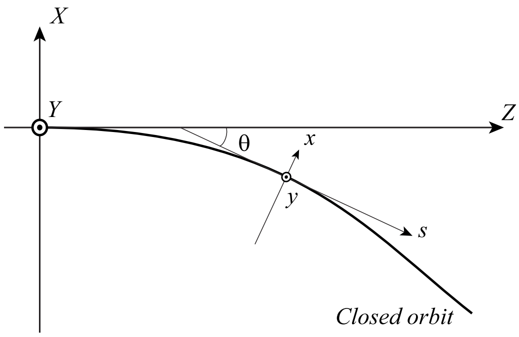

Fig. 1. Coordinate system. See Section 2.1 for details.

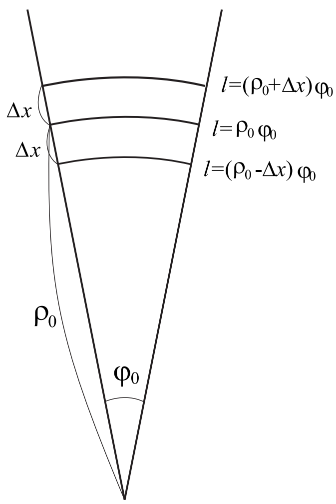

Fig. 2. Path length variation in an infinitely short sector bend segment. The path length l depends on the horizontal deviation, Δx.

closed orbit. The axis s points in the direction of the beam. The axis y is always parallel to the magnet axis Y while the other axes are rotated by the angle between Z and s axes, θ. The sign of θ is defined such that the projection of the axis s to the axis X is pointing to negative X when the angle is positive.

It is convenient to set the origin of the X–Y–Z system to a point where the axis s coincides with the axis Z, i.e. $\theta = 0$ there. Such a point is uniquely determined once the axis Z is defined unless the vertical field component B_Y alters as in undulators, for example. Without loss of generality, we employ a symmetric LGB, i.e. $\theta = 0$ in the middle of LGB, which can be the origin, and $B_Y(Z) = B_Y(-Z)$.

2.2. Multipole expansion

The magnetic field is generally expanded into Taylor series, and the multipole components are directly related to the series terms one by one. We use the following definitions throughout the paper:

$$B_y = \sum_{n=0}^{\infty} \frac{B_n x^n}{n!}, \tag{3}$$

and the corresponding multipole strengths are

$$K_n = \frac{B_n}{B\rho}, \tag{4}$$

where $B\rho$ is the magnetic rigidity of the beam. The number of poles is $2(n+1)$, i.e. $n = 0$ for the dipole field.

The above expansion is defined in the moving coordinate system since the magnetic field acts on the beam particle following the closed orbit with small transverse deviation.

Let us take an arbitrary location on the closed orbit, which we denote by $\left(X_0, Z_0\right)$. The two coordinate systems are then connected as

$$Z - Z_0 = x \sin\theta_0, \tag{5}$$

and

$$X - X_0 = x \cos\theta_0, \tag{6}$$

where θ_0 is the angle between the Z and s axes at $\left(X_0, Z_0\right)$. Hence we get

$$B_0 = B_Y\left(X_0, Z_0\right) \tag{7}$$

$$B_1 = \left.\frac{\partial B_Y}{\partial X}\right|_{X_0,Z_0} \cos\theta_0 + \left.\frac{\partial B_Y}{\partial Z}\right|_{X_0,Z_0} \sin\theta_0, \tag{8}$$

$$B_2 = \left.\frac{\partial^2 B_Y}{\partial X^2}\right|_{X_0,Z_0} \cos\theta_0 + \left.\frac{\partial^2 B_Y}{\partial Z^2}\right|_{X_0,Z_0} \sin\theta_0, \tag{9}$$

$$B_3 = \left.\frac{\partial^3 B_Y}{\partial X^3}\right|_{X_0,Z_0} \cos\theta_0 + \left.\frac{\partial^3 B_Y}{\partial Z^3}\right|_{X_0,Z_0} \sin\theta_0, \tag{10}$$

and so on. The first terms in B_n $(n > 0)$ originate at the transverse gradient, and the second terms originate at the longitudinal gradient. It is shown here that the longitudinal gradient generates the components higher than dipole.

When the transverse gradient terms are zero, the magnetic field of the LGB is a "rectangular-bend-like" field, where the field contour lines are parallel to the axis X. A special case, the "sector-bend-like" field, where the contour lines are parallel to the axis x is discussed later, although the rectangular-bend-like magnet may be preferable from the manufacturing point of view.

2.3. Feed-up

The so-called *natural focusing* in the horizontal plane comes from the geometric nature of sector bend magnets. It is naively expected that a focusing is due to a transverse gradient. However, in a sector bend magnet, the path length of a particle traveling off closed orbit is longer or shorter than that of the ideal particle on the closed orbit. The bending angle depends on the particle path, and thus the natural focusing arises from a pure dipole field. This applies to the quadrupole component as well, i.e., the quadrupole component included in a sector bend generates sextupolar focusing. The quadrupole component discussed here originates not only from the transverse gradient but also from the longitudinal one. Feed-up refers to this process whereby an nth term generates an $(n+1)$th term.

We now discuss an infinitely short segment of a sector bend magnet to formulate the "feed-up" described above. For the short segment, the vertical field, B_Y, is constant along s but depends on x.

As depicted in Fig. 2, the path length of a particle along the segment is

$$l = \left(\rho_0 + x\right)\varphi_0, \tag{11}$$

where ρ_0 is the bending radius and φ_0 is the bending angle of the segment for the particle on the closed orbit. For the off-closed-orbit particles, the deflection angle is

$$\begin{aligned}
\varphi &= B_Y l \\
&= \left(B_0 + B_1 x + \frac{B_2}{2}x^2 + \frac{B_3}{6}x^3 + \cdots\right)\left(\rho_0 + x\right)\varphi_0 \\
&= B_0\rho_0\varphi_0 \\
&\quad + B_0\varphi_0 x + B_1\rho_0\varphi_0 x \\
&\quad + B_1\varphi_0 x^2 + \frac{1}{2}B_2\rho_0\varphi_0 x^2 \\
&\quad \frac{1}{2}B_2\varphi_0 x^3 + \frac{1}{6}B_3\rho_0\varphi_0 x^3 \\
&\quad + \cdots
\end{aligned} \tag{12}$$