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Quantum-mechanical analysis of low-gain free-electron laser oscillators

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ABSTRACT

In the previous classical theory of the low-gain free-electron laser (FEL) oscillators, the electron is described as a point-like particle, a delta function in the spatial space. On the other hand, in the previous quantum treatments, the electron is described as a plane wave with a single momentum state, a delta function in the momentum space. In reality, an electron must have statistical uncertainties in the position and momentum domains. Then, the electron is neither a point-like charge nor a plane wave of a single momentum. In this paper, we rephrase the theory of the low-gain FEL where the interacting electron is represented quantum mechanically by a plane wave with a finite spreading length (i.e., a wave packet). Using the concepts of the transformation of reference frames and the statistical quantum mechanics, an expression for the single-pass radiation gain is derived. The spectral broadening of the radiation is expressed in terms of the spreading length of an electron, the relaxation time characterizing the energy spread of electrons, and the interaction time. We introduce a comparison between our results and those obtained in the already known classical analyses where a good agreement between both results is shown. While the correspondence between our results and the classical results are shown, novel insights into the electron dynamics and the interaction mechanism are presented.

1. Introduction

In the free-electron lasers (FELs) process, the low-quality of the electron beam results in a low-gain in radiation power at which the ratio of the output to the input field intensity is linear. In the low-gain regime [1–5], it is necessary to use an optical cavity to generate high brightness radiation after some round-trips of the radiation in the cavity. As a consequence of the lack of suitable mirrors for short-wavelength radiation such as x-rays, the light amplification should be achieved in a single pass through a very long undulator magnet. Therefore, high-gain FEL amplification is required within a single passage of a high-quality electron beam through a long undulator [6–8]. In this regime, the radiation power grows exponentially along the undulator. The low-gain FEL oscillators have been experimentally demonstrated in the infrared and ultraviolet (UV) regions [2,3]. On the other hand, it is widely known that the high-gain Self-Amplified Spontaneous Emission FEL (SASE FEL) is a potential candidate for the production of x-ray FELs [6]. Recently, it has been proposed that an x-ray FEL is also feasible in an oscillator configuration [9,10].

In Ref. [11], Madey first described the small-signal FEL gain using a quantum mechanical treatment where he used the Weizsacker–Williams method to calculate the quantum transition rates. Since the publication of this notable paper, many other authors proved that the FEL can be treated classically [12,13]. From a classical point of view, an electron considered as a point-like particle executes transverse oscillations that stimulate a coupling to the radiation fields through the Lorentz force [12,14,15]. In the previous quantum treatments of the low-gain regime [16–20], the electron is described by a plane wave extended infinitely along the electron bunch, being a single momentum state. In the physical world, the electron has no exact description, but has quantum uncertainties in the position and momentum. In this paper, the electron is considered as a wave packet (i.e., plane wave with a finite spreading length). This model of the electron is more rigorous and formally correct than the previous problematic models, the classical model of the point-like particle or the quantum model of the infinite plane wave. Using the wavepacket model of the electron, we present different insights on the electron dynamics and the interaction mechanism. The connections between our results and those of the classical approach are discussed. It is noted that in the quantum theory of high-gain regime FELs, the electron wave is assumed to be sufficiently extended over the group of bunched electrons due to the collective effect among electrons [21–23]. However, in this study, we focus on

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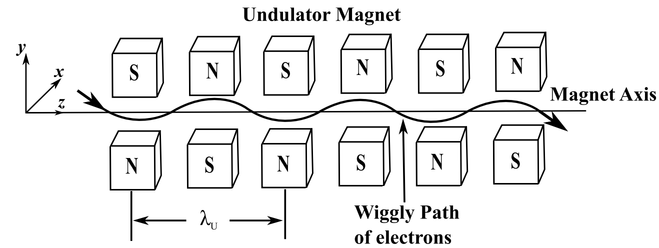

Fig. 1. Schematic view of a planar undulator magnet.

the low-gain regime of FELs where the case of the high-gain regime is beyond the scope of this work.

In the low-gain regime of FEL, the bunching process is weak and the single-particle treatment is applied. In this regime, the electrons are assumed to be randomly distributed under the influence of Coulomb repulsion forces between electrons. Since each electron is separated from the surrounding electrons, the wave function of electrons cannot be overlapped. Then, in the present work, the electron wave is assumed to be finite in length, a wavepacket with a finite spreading length ℓ. The interaction between the electron and the radiation field is achieved when the phase of the electron wave synchronizes with the phase of the radiation field. The spreading length of an electron wave is termed as the coherence length over which the phases of the single electron wave and the field are synchronized. Then, this coherence length almost corresponds to the length of the emitted radiation pulse. Another physical interpretation of the length of the electron wave can be given as follows. Assuming isotropic separation among neighboring electrons, an electron can be approximately represented by a plane wave confined in a box whose volume is ℓ^3 [24,25]. In principle, the uncertainty in the electron wave dimension, the length ℓ, is unknown and should be obtained experimentally. However, *the maximum possible length* of the electron wave $\ell_{\max}$ should be determined by the separation distance between the electron and its neighboring electrons during the interaction process. The maximum length of the electron wave $\ell_{\max}$, or the inter-particle distance, is approximately related to the average electron density N_t as $N_t \sim 1/[\ell_{\max}]^3$. It is noticed that the later relation between the inter-particle spacing and the electron density is commonly used in the weakly coupled plasma where the coulomb's repulsive forces are dominant.

Given these considerations, the quantum treatment based on the above description of electron is valid as long as the coherence length of the electron wave ℓ is larger than the radiation wavelength. The finite length of the electron wave causes a momentum uncertainty and then introduces a broadening to the spectrum of FEL radiation. In our model, the interaction is interpreted by the coupling between electron and EM waves and is realized at the synchronization between the phases of both waves. Therefore, the spreading length of the electron wave is understood as the radiation pulse length representing the classical relation of the normalized spectral FWHM $\Delta\omega/\omega = 1/N_U$ where N_U is the number of undulator periods. The relaxation and interaction times are conjugated with energy uncertainties and produce another broadenings to the radiation spectrum. As will be shown in Section 4, when the momentum uncertainty is neglected or assuming the electron is a point charge, the FEL behavior is described in the time domain and the previous classical results are attained from our results.

In this paper, the small-signal low-gain of FEL is calculated on the basis of a quantum mechanical model for electrons. In Sections 2 and 3, the dynamics of the EM wave and electron, respectively, are discussed. The EM wave is described classically and the electron is represented to be a plane wave with a finite spreading length. By choosing an appropriate moving frame [16], Maxwell's equations for the EM wave and the Schrödinger equation for electron wave are used to derive an expression for calculating the single-pass radiation gain. The expectation values are calculated by means of the density matrix method considering the statistical nature of electrons. For generality, we take into account the effect of electron relaxation that corresponds to the energy spread and is represented by phase distortion in the electron wave. In Section 4, the radiated power and the validity of our model are shown. In Section 4, the expression of the gain obtained in the moving frame is then transformed back to the laboratory frame using the relativistic Lorentz transformations. The analysis indicates that the spectral line of radiation is generally determined by the spreading length of the electron wave, the relaxation time, and the interaction time. The compatibility and correspondence of our analysis with the classical analyses are given. In Section 5, the conclusion is provided.

2. The dynamic of the laser field in the moving frame

The configuration of the electron beam and undulator magnets in an FEL is shown in Fig. 1. For a one-dimensional undulator field, the undulator axis is aligned with the z-axis, and the magnetic field is aligned vertically along the y-axis. The magnets have alternating poles and the direction of the magnetic field is reversed every undulator period λ_U.

The generation of coherent FEL radiation in the optical and x-ray regimes is achieved by using high-energy electron beams (i.e., the Lorentz factor $\gamma \gg 1$). The relativistic dynamics is then necessary to carry out the calculations in the laboratory frame. In the current analysis, we apply the Lorentz transformations to a moving frame in which a simplified unrelativistic treatment is used [16]. From here on, we will use an arbitrary moving frame with a fixed velocity $\mathbb{V}$ which is slightly less than the speed of light c. Once we get the expression for the radiated power in the moving frame, the results are then transformed back to the laboratory frame using Lorentz transformations.

Looking at the interaction in a frame moving with a velocity $\mathbb{V}$, the laser wavelength in the moving frame $\acute{\lambda}$ relates to that in the laboratory frame λ by

$$\acute{\lambda} = \lambda \frac{[1 + (\mathbb{V}/c)]}{\sqrt{1-(\mathbb{V}/c)^2}}. \tag{1}$$

Therefore, the laser frequency in the moving frame $\acute{\omega}$ is transformed by

$$\acute{\omega} = \omega \frac{[1 - (\mathbb{V}/c)]}{\sqrt{1-(\mathbb{V}/c)^2}} = \acute{\beta}c. \tag{2}$$

The evolution of the transverse electric field of the laser $\acute{E}_{xL}$ is described by the wave equation

$$\nabla^2 \acute{E}_{xL} - \mu_0\varepsilon_0 \frac{\partial^2 \acute{E}_{xL}}{\partial t^2} = \mu_0 \frac{\partial \acute{J}_x}{\partial t}, \tag{3}$$

where $\acute{J}_x$ is the x-component of the current density. In the low-gain regime of FELs, the collective effects of space charge do not play an important role where the beam current is significantly small. Thus, the space charge density is neglect in Eq. (3).

The laser field is assumed to be a polarized plane wave in the x-direction. Therefore, the electric field is assumed to take the form

$$\acute{E}_{xL} = \acute{F}(\acute{t},\acute{z}) T_x(x,y) e^{j(\acute{\omega}\acute{t} - \acute{\beta}\acute{z})} + c.c., \tag{4}$$

$\acute{F}(\acute{t},\acute{z})$ is the field amplitude and $T_x(x,y)$ is the transverse field distribution. $T_x(x,y)$ satisfies the relation of

$$\left(\nabla^2 + \mu_0\varepsilon_0\acute{\omega}^2\right) T_x(x,y) e^{-j\acute{\beta}\acute{z}} = 0, \tag{5}$$

and is normalized assuming

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |T_x(x,y)|^2 dx dy = 1. \tag{6}$$

The transverse field distribution T_x is almost constant in the electron beam and is taken into account to evaluate the coupling efficiency between the electron beam and the laser field.