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Unfolding and unfoldability of digital pulses in the z-domain

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A B S T R A C T

The unfolding (or deconvolution) technique is used in the development of digital pulse processing systems applied to particle detection. This technique is applied to digital signals obtained by digitization of analog signals that represent the combined response of the particle detectors and the associated signal conditioning electronics. This work describes a technique to determine if the signal is unfoldable. For unfoldable signals the characteristics of the unfolding system (unfolder) are presented. Finally, examples of the method applied to real experimental setup are discussed.

1. Introduction

In radiation spectroscopy, the development of Digital Pulse Processing is usually focused on direct synthesis of pulse shapes using digitized signals coming from particle detection used in radiation measurement systems [1,2]. The ideal shaping for a given detector depends on the shape of the Digital Pulse Processing (DDP) system input signal and the associated noise characteristics [3]. Thus, specific techniques are used to synthesize various shapes to maximize their Signal-to-Noise Ratio [4–6] or to minimize the effect of ballistic deficit or to reduce the pulse pile-up [1].

A subset of Digital Pulse Processing is the unfolding (or deconvolution) technique that allows the transformation of the digitized signal into a unit impulse in the discrete-time domain (see [7] and the references therein). The unfolding technique can be applied to linear pulse processing systems that are either time-invariant or time-variant. A detection system that uses this technique usually includes the unfolding of the digital signals into unit impulses, followed by the synthesis of digital signal processing systems with unit impulse responses equivalent to the desired pulse shape.

In this paper, we describe a technique to determine if a pulse shape can be unfolded (unfoldability), and in such case, a method that allows the synthesis of its unfolder, either exactly or as a close approximation. The proposed method is suitable for real-time implementation.

2. Unfolding and unfoldability

In general, digital unfolding systems have a unit impulse response $h[n]$ whose convolution with the input signal $x[n]$ produces a unit impulse $\delta[n]$ as explained in [8]

$$x[n] * h[n] = \delta[n-d],\ d \in \{0, 1, 2, \dots\} \tag{1}$$

where d is the delay of the unit impulse in cycles.

Since Eq. (1) is a convolution, in the z-domain, it can be presented as follows

$$X(z) \cdot H(z) = z^{-d} \tag{2}$$

Therefore, the shaper that unfolds the pulse is equal to

$$H(z) = \frac{z^{-d}}{X(z)} \tag{3}$$

On the other hand, when the z-transform is applied to $x[n]$, the arrangement of its poles and their zeros are obtained. It is also known that systems are stable when all its poles are inside the Region Of Convergence (ROC) (i.e. $z < 1$), oscillating when at least one of its poles is at the circle $z = 1$ and unstable when at least one of its poles is outside the ROC (i.e. $z > 1$).

When $d = 0$, according to (3), $H(z)$ is the inverse of $X(z)$. It implies that the zeros of $X(z)$ are the poles of $H(z)$ and vice versa. In addition, $H(z)$ must be stable. Therefore, for a signal $X(z)$ to be unfoldable, both its zeros and poles must be within the ROC (i.e. $z < 1$).

When $d > 0$, $X(z)$ is delayed by d cycles, so the result $X(z) * H(z)$ must be a unit impulse delayed by d cycles (i.e. $z^{-d}\delta(z)$). Adding a delay of a certain number of cycles implies the inclusion of the same number of poles in $H(z)$ at $z = 0$. These poles have no effect on the stability of $H(z)$ but their inclusion may be mandatory to convert a non-casual unfolder obtained into a casual one by applying (3). As very simple example,

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Table 1
Unfolder characteristics. Non-causal unfolders are not implementable, but they can be solved by adding d grades in the denominator and thus shifting by d cycles the unit impulse as explained in text.

Unfolder characteristic	Input signal characteristic	Consequence in the unfolder
IIR	Zeros at $z \neq 0$	Poles at $z \neq 0$
Oscillating, potentially unstable	$\exists$ zeros at $\lvert z\rvert = 1$	$\exists$ poles at $\lvert z\rvert = 1$
Unstable	$\exists$ zeros at $\lvert z\rvert > 1$	$\exists$ poles at $\lvert z\rvert > 1$
Non-causal	Grad num < grad den	$H(z)$: Grad num > grad den

if $X(z) = \frac{1}{z-0.5}$, its unfolder is $H(z) = z - 0.5$ which is non-casual. To convert $H(z)$ into casual it must be delayed by one (or more) cycles, that is $H(z) = \frac{z-0.5}{z}$ whose convolution with $X(z)$ gives a unit impulse delayed one pulse.

Eq. (3) has solution only for signals whose poles and zeros are inside the ROC (e.g. exponential and $(\mathrm{RC})^n$ pulses). In contrast, whenever a shape is symmetric (e.g. trapezoidal, triangular or cusp-like), their zeros are located at $|z| = 1$. Consequently, its unfolder has their poles located at $|z| = 1$ and the unfolder is oscillating or potentially unstable. Fortunately, pulses coming from a radiation detector are rarely symmetric. In Table 1 the characteristics of the unfolder $H(z)$ as function of the input signal $X(z)$ are listed.

It is known that convolution in time-domain is equal to multiplication in the z-domain. Thus, when two signals are convoluted in time-domain it is equivalent to join all their zeros and poles. As mentioned previously, the placement of their zeros indicate when signals are unfoldable. Therefore, the result of the convolution of two unfoldable signals is also unfoldable. In contrast, the result of the convolution of an unfoldable signal and a non-unfoldable signal is non-unfoldable. By last, the addition of non-unfoldable signals are also non-unfoldable. These facts are always valid unless the two combined signals cancel out each of their zeros reciprocally. In this case, a new analysis have to be carried out.

3. Examples

3.1. Unfolding of exponential pulses

In the discrete-time domain, a generic exponential pulse can be defined as

$$x[n] = A \cdot \exp\left(\frac{-n}{\tau}\right) \tag{4}$$

where τ is the decay constant. In the z-domain, it becomes

$$X(z) = \frac{z}{z-a} \tag{5}$$

where

$$a = \exp\left(\frac{-\Delta T}{\tau}\right) \tag{6}$$

and ΔT is the sample period of the digitized signal.

Applying the exposed method, we obtain the following unfolder with no delay (i.e. $d = 0$)

$$H(z) = \frac{z-a}{z} \tag{7}$$

The impulse response in time-domain and pole-zero maps of $X(z)$, $H(z)$ and $Y(z)$ are shown in Fig. 1. This result agrees with that shown in [7] for exponential pulses.

3.2. Sum of exponential pulses

As stated in [7] and according to the explanation given in Section 2, the unfolding of an exponential pulse can be extended to additions of exponential pulses.

Using the linearity property of the z-transform, the sum of two exponential pulses can be expressed in the z-domain as

$$X(z) = X_a(z) + X_b(z) = \frac{Az}{z-a} + \frac{Bz}{z-b} \tag{8}$$

where A, B are their amplitudes and a, b are their delay constants. Disregarding A and B, which do not affect the stability of the system, the equation can be rewritten in the following way

$$X(z) = \frac{z((z-a)+(z-b))}{(z-a)(z-b)} \tag{9}$$

Recall that for the system to be unfoldable, both poles and zeros must be within the ROC region. Clearly, the poles of $X(z)$ are the poles of $X_a(z)$ and the poles of $X_b(z)$. All the poles of $X(z)$ will be within the ROC if those of $X_a(z)$ and $X_b(z)$ are too. With respect to the zeros, the system has one at $z = 0$ and another at $z = (a+b)/2$, so if $a, b < 1$, the zeros will also be within the ROC.

In general, (9) can be extended to an arbitrary number of exponentials and it is trivial to demonstrate that $X(n)$ is unfoldable whenever their decay constants are below 1. Therefore, we can conclude that the sum of exponential pulses are unfoldable.

In the case where one of the pulses is delayed with respect to the others, this affirmation cannot be always true since new poles are added and they can make the system unstable or oscillating. In Fig. 2 an oscillating unfolder shaper is shown. The input signal is the sum of two exponential signals with $a = b = 0.8$, one of them is delayed by one cycle.

3.3. Convolution of exponential pulses and $(\mathrm{RC})^n$ pulses

As exposed in Section 2, the convolution of exponential pulses in time-domain is equivalent to multiplication in z-domain. Thus, the effect of convolving signals is to add new poles and zeros without displacing the original ones. Therefore, because exponential pulses are unfoldable, the convolution of exponential pulses are unfoldable.

An arbitrary convolution of exponential pulses gives rise to $(\mathrm{RC})^n$ pulses. This pulse can be represented in the z-domain as

$$X(z) = \frac{z^2}{(z-a)^2} \tag{10}$$

Applying the exposed method, we obtain the following unfolder with no delay (i.e. $d = 0$):

$$H(z) = \frac{(z-a)^2}{z^2} \tag{11}$$

The impulse response in time-domain and pole-zero maps of $X(z)$, $H(z)$ and $Y(z)$ are shown in Fig. 3. This result also agrees with that shown in [7] for exponential pulses.

3.4. Derivatives and integrals of unfoldable signals

It is known that given an input signal $X(z)$, its n-derivative is $\left(\frac{z-1}{z}\right)^n X(z)$ whereas its n-integral is $\left(\frac{z}{z-1}\right)^n X(z)$. In both cases $X(n)$ is multiplied by $\left(\frac{z-1}{z}\right)^n$ (n in case of the n-derivative and $-n$ in case of the n-integral).

Thus, given a stable signal $X(z)$, $\left(\frac{z-1}{z}\right)^n X(z), n \in \mathbb{Z}$ will be also stable if either poles and zeros of $\left(\frac{z-1}{z}\right)^n$ are within the ROC. In case of integrals ($n \leq -1$), the poles are located in $z = 1$ being able to make the system oscillating or unstable depending on $X(z)$. However, due to the fact that $\left(\frac{z-1}{z}\right)^n$ itself is unfoldable (see Section 2), we can conclude if a signal is unfoldable, its n-derivative or n-integral is also unfoldable.