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Correlation methods for the analysis of X-ray polarimetric signals

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A B S T R A C T

X-ray polarimetric measurements are based on studying the distribution of the directions of scattered photons or photoelectrons and on the search of a sinusoidal modulation with a period of π. We developed two tools for investigating these angular distributions based on the correlations between counts in phase bins separated by fixed phase distances. In one case we use the correlation between data separated by half of the bin number (one period) which is expected to give a linear pattern. In the other case, the scatter plot obtained by shifting by 1/8 of the bin number (1/4 of period) transforms the sinusoid in a circular pattern whose radius is equal to the amplitude of the modulation. For unpolarized radiation these plots are reduced to a random point distribution centred at the mean count level. This new methods provide direct visual and simple statistical tools for evaluating the quality of polarization measurements and for estimating the polarization parameters. Furthermore they are useful for investigating distortions due to systematic effects.

1. Introduction

Measurements of the linear polarization in the X-ray band are based on the detection of anisotropies either in the angular distribution of scattered photons or in the initial direction of photoelectrons (for a comprehensive review see the book by Fabiani and Muleri [1]). The latter technique appears now more promising with the development of the Gas Pixel Detector (hereafter GPD) [2–4] that will be the focal plane instrument of the first polarimetric mission after the pioneering age of OSO 8. Indeed IXPE (Imaging X-ray Polarimetry Explorer, Weisskopf et al. [5]) has been selected as the next SMEX NASA mission for a flight in late 2020. With three mirrors and three GPDs it will perform spectral–temporal–angular resolved polarimetry as a break-through measurement in astrophysics, allowing the opening of a new window on the high-energy sky. Meanwhile XIPE (X-ray Imaging Polarimetry Explorer) [6], has completed its phase A study in the framework of the 4th ESA call for a medium size mission.

In the present paper we show some simple correlation methods useful for the detection and for the parameter estimates of a polarized signal. These methods are based on the phase distribution histograms of photons (or electrons) and do not require measurements of the Stokes' parameters; for this reason they have the advantage of simplifying the statistical tools used for evaluating the significance of the polarimetric estimates [7]. Our new method is therefore complementary to the classical Stokes' analysis.

This topic has been previously studied by different authors. The relation between sensitivity and significance of a polarization measurement [8,9] has been revisited by means of simulation [10]. The use of Stokes' parameters for X-ray polarimetry has been explored by Kislat et al. [11] while the possibility to extend the standard tools of X-ray astronomy (XSPEC) to polarization by using Stokes parameter has been studied very recently by Strohmayer [12]. Other authors, instead, investigated the possibility of using Bayesian methods to derive the polarization angle and degree, which may be interesting when low signal-to-noise ratio are expected [13,14].

An important issue in polarization measurements is the possibility to have a noise distribution differing from the Poissonian one because of intrinsic effects in the detection technique. For instance, in a photoelectric polarimeter as the GPD the uncertainty on the initial direction of the photoelectron can be influenced by the hexagonal geometrical pattern of pixel plane (especially for tracks comprising a very small number of pixels) producing a not uniform spread of angular directions in the phase histograms [15].

Thus the assumption that the noise on the angular distribution is white and purely Poissonian cannot be always verified. Our correlation methods are not based on this assumption and provide direct visual

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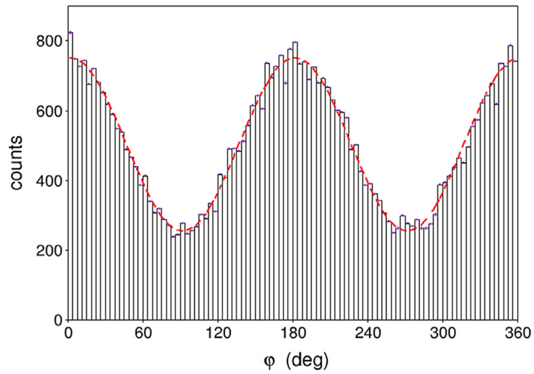

Fig. 1. Histogram of the angular distribution of the initial direction of electrons due to a polarized X-ray flux. The dashed curve is the best fit modulation.

tools for evaluating the quality of polarization measurements and for investigating distortions due to systematic effects.

2. Polarization data

For each detected photon the information on its linear polarization is given by the azimuthal angle φ measured from the direction of a scattered photon (for scattering polarimeters) or from the initial direction of the photoelectron trajectory (for photoelectric polarimeters). The detection of a polarization is therefore obtained from the angular distribution of the φ values of a number N of observed photons and particularly from the amplitude of a sinusoidal modulation with a period of π (or 180°), due to the differential cross section of the physical process involved (Compton scattering and photoelectric absorption, respectively). In the following we will refer to this histogram as the data set $\{n_k | k = 1, \ldots, M\}$, with a total bin number M and a number of events in the kth bin indicated by n_k. One can write the expected modulation curve as

$$n(\varphi_k) = A \ \sin(2(\varphi_k + \psi)) + \langle n \rangle \quad , \tag{1}$$

where $\varphi_k = (2\pi/M)(k - 1/2)$ is the phase of the bin centre, A the amplitude of the modulation and ψ is the polarization angle; for a total number of photons N one obviously has

$$\langle n \rangle = N/M \quad . \tag{2}$$

In Fig. 1 the histogram of a modulation curve of experimental data for a totally polarized source is shown. The polarization degree p of the incoming signal is then evaluated by

$$p = \frac{1}{\mu} \frac{A}{\langle n \rangle} \quad , \tag{3}$$

where μ is the instrumental modulation factor, i.e. the amplitude resulting from a totally polarized input radiation. The noise is due to the statistical fluctuations of counts and to possible instrumental effects.

As it will be clear in the following it is advantageous to consider M as a multiple of 8 (in the following examples we will consider bin numbers with this property).

3. The method of the linear correlation plot

We create a scatter plot with $M/2$ points having coordinates $(n_k, n_{k+M/2})$ $(1 \le k \le M/2)$. If a polarization modulated signal is present, these point will be distributed along a straight segment with an inclination of 45° and with a length equal to the double of the modulation amplitude. The linear correlation coefficient r can provide useful information for estimating the statistical S/N ratio. The significance of the polarization measurement is obtained from the likelihood to have a corresponding r with $M/2$ degrees of freedom. The linear plot for the same data set in Fig. 1 is given in Fig. 2 (left panel), where the values of the linear correlation coefficient and of the best fit line parameters are also reported together with the statistical uncertainties:

$$n_{k+M/2} = h + s\ n_k \tag{4}$$

are also reported.

For a modulation as the one of Eq. (1) the expected values of the slope s is 1 and of the constant h is zero and it is actually found within 1 standard deviation as shown in Fig. 2 (left panel). In the case of a non polarized radiation we expect no correlation between the phase bins and in the plot the corresponding points will be randomly distributed around the mean value, with a linear correlation coefficient close to zero.

It is well known from the correlation theory that r^2 is the fraction of the variance *explained* by the linear regression and that $1 - r^2$ is proportional to the *residual* variance due to the noise; thus the simplest way to evaluate the S/N ratio of the polarization is given by $\sqrt{r^2/(1-r^2)}$. We stress that this simple formula is independent of any assumption of the nature of the noise and particularly if it is only due to the Poisson statistics or to the occurrence of possible systematic effects. In particular values of s and/or h non consistent with 1 and zero, respectively, likely indicate a systematic deviation.

A linear correlation with unit slope results not only from a sinusoidal pattern like that of Eq. (1), but also from any distribution with a period $M/2$, and thus this plot is not useful for detecting a polarized signal itself, but only for estimating the S/N ratio and the occurrence of some systematic effects. These in fact can introduce changes in the amplitude of the angular distribution resulting in a value of the slope s different from unity. For instance, systematic deviations affecting an odd harmonic, like the 3rd one, which are anticorrelated and therefore the expected value is $s = -1$, would produce a decrease of the slope (see Section 7).

4. The method of the circular plot

A useful tool for measuring the polarization parameters and for a quick inspection of data is based on the scatter plot of the histogram of the angular distribution of events in the (ξ, η) space, where these quantities are the histogram bin counts with the latter one shifted by $M/8$ bins. Consider the distribution of M points having the coordinates $(\xi_k = n_k, \eta_k = n_{k+M/8})$, $(1 \le k \le 7M/8)$, completed with the $M/8$ points $(\xi_k = n_k, \eta_k = n_{k-7M/8})$, $(7M/8 + 1 \le k \le M)$. If the counts are distributed with a $\sin(2\varphi)$ modulation the phase difference of $M/8$ corresponds to 1/4 of the period and the points in the (ξ, η) space will be distributed in a circular pattern having the radius equal to the modulation amplitude. Thus the problem of detecting a polarization is then reduced to that of finding such a circular distribution of data.

4.1. Measure of the polarization degree and angle

For each point of the plot we can compute the corresponding radius R_k

$$\begin{aligned} R_k &= \sqrt{(n_k - \langle n \rangle)^2 + (n_{k+M/8} - \langle n \rangle)^2} \\ &= \sqrt{(\xi_k - N/M)^2 + (\eta_k - N/M)^2} \end{aligned} \tag{5}$$

The distribution of R_k against the phase bin number angle or the bin centre angle φ_k is expected to be consistent with a constant and the ratio between the mean value of the radius $\langle R \rangle$ and its standard deviation σ_R is an estimator of the S/N ratio. In Fig. 2 (right panel) it is reported the circular polarization plot for the same data set of the left panel. The mean radius is $\langle R \rangle = (249.20 \pm 2.8)$. The amplitude found by the usual method of a $\sin(2(\varphi - \psi))$ least-squares fitting results $A = (247.5 \pm 3.6)$, compatible with the previous estimate within 1σ.

Note that in the circular plot each value n_k is used two times: first for computing η and then for ξ; thus the M values distributed in the two