Nuclear Inst. and Methods in Physics Research, A 884 (2018) 119–127

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research, A

journal homepage: www.elsevier.com/locate/nima

# Multiplicity counting from fission detector signals with time delay effects

L. Nagy [a,b], I. Pázsit [a,*], L. Pál [c]

[a] Chalmers University of Technology, Department of Physics, Division of Subatomic and Plasma Physics, SE-412 96 Göteborg, Sweden
[b] Budapest University of Technology and Economics, Institute of Nuclear Techniques, H-1111 Budapest, Hungary
[c] Centre for Energy Research, Hungarian Academy of Sciences, H-1525 Budapest 114, POB 49, Hungary

A B S T R A C T

In recent work, we have developed the theory of using the first three auto- and joint central moments of the currents of up to three fission chambers to extract the singles, doubles and triples count rates of traditional multiplicity counting (Pázsit and Pál, 2016; Pázsit et al., 2016). The objective is to elaborate a method for determining the fissile mass, neutron multiplication, and ($\alpha$, n) neutron emission rate of an unknown assembly of fissile material from the statistics of the fission chamber signals, analogous to the traditional multiplicity counting methods with detectors in the pulse mode. Such a method would be an alternative to He-3 detector systems, which would be free from the dead time problems that would be encountered in high counting rate applications, for example the assay of spent nuclear fuel.

A significant restriction of our previous work was that all neutrons born in a source event (spontaneous fission) were assumed to be detected simultaneously, which is not fulfilled in reality. In the present work, this restriction is eliminated, by assuming an independent, identically distributed random time delay for all neutrons arising from one source event. Expressions are derived for the same auto- and joint central moments of the detector current(s) as in the previous case, expressed with the singles, doubles, and triples ($S$, $D$ and $T$) count rates. It is shown that if the time-dispersion of neutron detections is of the same order of magnitude as the detector pulse width, as they typically are in measurements of fast neutrons, the multiplicity rates can still be extracted from the moments of the detector current, although with more involved calibration factors. The presented formulae, and hence also the performance of the proposed method, are tested by both analytical models of the time delay as well as with numerical simulations. Methods are suggested also for the modification of the method for large time delay effects (for thermalised neutrons).



## 1. Introduction

A new method of multiplicity counting has recently been suggested by the present authors [1,2]. The method uses the fluctuations of the fission detector signals in the Campbelling (current) mode to extract the same information about the sample as the traditional multiplicity counting methods do from the $S$, $D$, and $T$ rates [3,4]. Instead of the counting statistics of discrete pulses, the new method is based on the first three central moments (cumulants) of the time-resolved signals of one or more (up to three) fission chambers. It was shown that with the use of the concept of superfission of Böhnel [5], and with the assumption of simultaneous detection of all neutrons originating from one source event, the same information can be extracted as from the multiplicity rates. This is partly because the traditional pulse counting based methods rely also on the same Böhnel model, and thus it was relatively simple to derive relationships between the moments of the detector current and the $S$, $D$ and $T$ count rates.

The interest in the new method was partially curiosity-driven, so as to provide an alternative to the existing multiplicity counting methods [6–9]. More important, it would also have the advantage of being free from the dead-time problem, and less sensitive to gamma background than organic scintillators [10,11]. In fission chambers, gamma detections generate a much narrower and smaller amplitude pulse than neutron detection, in contrast to organic scintillators.

However, the assumption that all neutron detections from one source emission are exactly simultaneous is not realistic. Even with the assumptions of the Böhnel model, i.e. that the release of the source neutrons from the item (i.e. a fissile assembly) takes a negligible time, also with internal multiplication being included, the actual detection process takes place with a certain time delay with respect to the instant of emission, due to the transport of neutrons between the source (fissile mass) and the detectors, and possible slowing down before detection. This latter is quite obvious with the thermal neutron based multiplicity counting systems, such as an AWCC (Active Well Coincidence Counter), where neutrons are first slowed down before detection. The slowing down time (time to detection) is a random parameter, which differs for each neutron. For such systems, with a good approximation, one can assume an exponential distribution of the detection times (the “detector die-away time” in safeguards parlance).

The random delay in the detection of neutrons of common origin in traditional multiplicity counting systems is actually not completely

* Corresponding author.
*E-mail address:* imre@chalmers.se (I. Pázsit).

https://doi.org/10.1016/j.nima.2017.12.028
Received 28 September 2017; Received in revised form 7 December 2017; Accepted 8 December 2017
Available online 13 December 2017

disadvantageous. Because of dead time effects, the same detector cannot detect two or more neutrons simultaneously, thus with one single detector, the non-simultaneous arrival/detection is an advantage (even if in practice this problem is handled by using an array of detectors with independent electronics). The way of accounting for the random (and non-simultaneous) detections is to have one of more gates open after a detection during a certain length of time, which is short enough for distinction between different source events, but covers a long enough fraction of the detector die-away time such that the probability of detection(s) from the same source event should not be too low. This of course means loss of coincident counts of neutrons from a single emission, and it is taken into account with the so-called doubles and triples gate factors $f_d$ and $f_g$, respectively. These are estimated with the assumption of the detector die-away time, and are both less than unity (in the corresponding orders of the ratio of the gate length to the die-away time).

The situation is different for the case of fission chambers. These have no limitations of the dead time type, and can in principle detect a large number of particles either simultaneously, or in a fast sequence. This fact in itself represents a significant advantage from the point of view of the detection process. On the other hand, the principle of multiplicity counting from fission chamber currents, in its form as published in our previous work [2], is based on the exact overlapping of the (multiple) pulses corresponding to neutrons from the same source emission event. As soon as the neutrons are not detected simultaneously, the pulses will not completely overlap, and the formulae derived in [2] will not be valid.

It can be intuitively expected, and will be shown rigorously in the derivations in the body of the paper that, if the width of the distribution of the arrival/detection times is comparable with the detector pulse width, it is still possible to extract the multiplicity rates from the detector currents, but some "correction factors" will appear in the formulae, similar in character to the doubles and triples gate factors of traditional multiplicity counting. This is the case when fast neutrons are detected directly from the sample, without slowing down. It is easy to show that with the energy spectrum of spontaneous fission, and corresponding velocity spread of fast neutrons, together with plausible distances between the fissile sample and the detectors, the fluctuations in the arrival times are comparable with the detector pulse width. If, on the other hand, the neutrons are slowed down, the spread in the arrival times will be so much larger than the detector pulse width, such that double or triple pulse overlappings will become extremely unlikely, and any higher order moment of the detector current will only depend on the singles rates. For this case the method in its present form (one-point (in time) distributions) is not applicable, therefore other methods (temporal correlations, i.e. two- or three-point distributions in time) will have to be applied. These will be briefly discussed at the end of the paper, but they lie outside the scope of the present work.

To account for the non-simultaneous detection of neutrons from the same source emission, in this work we will assume an independent, identical random distribution of the neutron arrival times to the detector. Exact analytical relationships will be derived for the three lowest order auto- and cross-cumulants of the signals of up to three fission chambers. It is shown that these are still explicitly related to the traditional multiplicity rates, but some modifying factors, similar to the gate factors of the traditional method, appear in them. Exact analytical formulae are given for these correction factors. This also means that as long as the correction factors are not negligibly small, the sample parameters still can be unfolded from these cumulants.

In the following, first the traditional formulae for the $S$, $D$ and $T$ count rates will be briefly recalled. Then the formalism, and the results for the cumulants of the detector current for the case of simultaneous detections will be cited from our previous work, in order to make the paper reasonably self-contained. After that, the time delay effects will be introduced into the formalism, and the first three auto- and cross-cumulants of up to three detectors will be derived. The resulting expressions will be then brought to a form similar to the previous ones which do not account for the time delay, but with the introduction of the equivalents of the gate factors. Arrival time distributions, due to the velocity distribution of fission neutrons, will be analytically derived, and the correction factors (gate factors) quantitatively calculated with some model detector pulse functions. The dependence of the gate factors on the distance between the fissile mass and the detector is determined quantitatively, suggesting the applicability of the method in fast neutron measurements.

## 2. Traditional multiplicity counting

For easy later reference, first the formulae of the traditional multiplicity counting method will be briefly summarised by using the terminology of [4]. The majority of source neutrons arise from spontaneous fission, with a fission source intensity $Q_f \equiv F$ fissions/s, which emits neutrons with a number distribution $p_{sf}(n)$, whose $k$th order factorial moments are denoted as $\nu_{sf,k}$. In addition, single neutrons are also emitted via $(\alpha, n)$ reactions, with an intensity $Q_\alpha$. This latter is eliminated from the formulae by introducing the alpha ratio $\alpha$ and the total source intensity $Q_s = F(1 + \alpha\nu_{sf,1})$ as usual. Following a source event, the emitted neutrons may cause induced fissions with a probability $p$ (it is assumed that capture is negligible), resulting in an induced fission event with emission number distribution $p_i(n)$ and factorial moments of order $k$ denoted as $\nu_{i,k}$, or escape the sample with a probability $1 - p$.

As is described in [5] and [12], a backward type master equation can be written down for the probability distribution $P(n)$, and its generating function, of the number of all neutrons emitted from the sample *per one source event*. From this equation, explicit solutions can be obtained for the first three factorial moments $\nu_k$ of this distribution (actually, for any order moments, as was demonstrated in Ref. [13]). From these moments of discrete (but unmeasurable) number distributions, one derives singles, doubles and triples detection rates, which are all proportional to the total source intensity $Q_s = F(1+\alpha\nu_{sf,1})$. It simplifies the notations to introduce the modified Böhnel moments

$$\tilde{\nu}_i = \nu_i(1 + \alpha\nu_{sf,1}). \tag{2.1}$$

By using the well-known expressions for the Böhnel moments [3,4], the first three of the $\tilde{\nu}_i$ are given as

$$\tilde{\nu}_1 = M\,\nu_{sf,1}(1 + \alpha) \tag{2.2}$$

$$\tilde{\nu}_2 = M^2\left[\nu_{sf,2} + \left(\frac{M-1}{\nu_{i1}-1}\right)\nu_{sf,1}(1+\alpha)\nu_{i2}\right] \tag{2.3}$$

and

$$\tilde{\nu}_3 = M^3\left[\nu_{sf,3} + \left(\frac{M-1}{\nu_{i1}-1}\right)\left[3\nu_{sf,2}\nu_{i2} + \nu_{sf,1}(1+\alpha)\nu_{i3}\right] + 3\left(\frac{M-1}{\nu_{i1}-1}\right)^2\nu_{sf,1}(1+\alpha)\nu_{i2}^2\right] \tag{2.4}$$

where $M$ is the leakage multiplication

$$M = \frac{1-p}{1-p\,\nu_{i1}}. \tag{2.5}$$

Using the above, the $S$, $D$ and $T$ rates of traditional multiplicity counting are written as (cf. [4])

$$S = F\varepsilon\,\tilde{\nu}_1; \qquad D = \frac{F\varepsilon^2 f_d\,\tilde{\nu}_2}{2}; \quad \text{and} \quad T = \frac{F\varepsilon^3 f_t\,\tilde{\nu}_3}{6}, \tag{2.6}$$

where $\varepsilon$ is the detection efficiency, and $f_d$ and $f_t$ are the doubles and triples "gate factors", mentioned earlier. The detection efficiency accounts for the fact that not all neutrons, impinging on the detector, will be actually detected, whereas the gate factors quantify that only a fraction of the physical detections, namely those taking place within the gate time used in the measurement, will contribute to the double and triple coincidence counts. A discussion on the gate factors and their