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Statistical fluctuations in cooperative cyclotron radiation

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A B S T R A C T

Shot noise is the cause of statistical fluctuations in cooperative cyclotron radiation generated by an ensemble of electrons oscillating in magnetic field. Autophasing time – the time required for the cooperative cyclotron radiation power to peak – is the critical parameter characterizing the dynamics of electron-oscillators interacting via the radiation field.

It is shown that premodulation of charged particles leads to a considerable narrowing of the autophasing time distribution function for which the analytic expression is obtained. When the number of particles N_e exceeds a certain value that depends on the degree to which the particles have been premodulated, the relative root-mean-square deviation (RMSD) of the autophasing time δ_T changes from a logarithmic dependence on N_e ($\delta_T \sim 1/\ln N_e$) to square-root ($\delta_T \sim 1/\sqrt{N_e}$).

A slight energy spread (∼4%) results in a twofold drop of the maximum attainable power of cooperative cyclotron radiation.

1. Introduction

A considerable attention has been paid recently to the generation of short superradiance pulses [1,2], also termed as "self-amplified spontaneous emission" or "cooperative radiation" [3], using short electron beams propagating in complex electrodynamical structures (undulators, resonators, dielectric and corrugated waveguides, or photonic crystals) [1,2,4–6]. The possibility to produce short pulses of cooperative radiation was first substantiated in [7,8] for free electron lasers (FELs). However, the experimentally measured output characteristics of cooperative radiation (power, radiation spectra, and energy in a pulse) have seemed really stochastic. According to theoretical [9,10] and experimental [11–15] studies, statistical properties of cooperative radiation depend on the shot noise which is inherent to charged-particle beams. It is noteworthy that frequency and time stability of the radiation pulse is crucial for many applications. For this reason, electron beams are premodulated at radiation frequency to reduce the influence of shot noise on cooperative radiation [16–20].

In a microwave range, short-pulse cyclotron resonance masers (CRMs) [21–23] and Cherenkov generators [24–26] are more commonly used. The generators of cooperative radiation operate in two regimes: traveling wave [24–26] and backward wave [27–31]. The distinguishing feature of the traveling-wave regime is that the group velocity of electromagnetic waves and the velocity of charged particles are co-directional. This regime was applied in the first experiments with short-pulse CRMs [22,23] and Cherenkov generators [24]. Theoretical consideration of radiation gain in short-pulse traveling-wave generators revealed an important peculiarity: the stability of the cooperative radiation parameters can be improved noticeably by injecting into the electrodynamical structure of a beam with a sharp front whose duration is comparable with the wave period [25,26]. In this case, the Fourier transform of the beam current contains quite a significant spectral component at radiation frequency. As a result, the generation of electromagnetic oscillations starts with coherent spontaneous emission of the whole beam instead of incoherent spontaneous emission from individual particles. As a result, the degree of fluctuations in cooperative radiation is decreased.

Though the early short-pulse CRMs and Cherenkov generators operated in the traveling-wave regime, the most impressive results were obtained in experiments with short-pulse backward-wave tubes generating cooperative radiation whose peak power was appreciably greater than the beam power [29–31]. For example, the peak radiated power of 1.2 GW was attained at 9.3 GHz for beam power of 0.87 GW [29]. The stability of output parameters of the cooperative-radiation pulse generated in the backward-wave regime, as in a short-pulse traveling-wave generator, strongly depended on the beam front.

Analysis of theoretical and experimental works on generation of cooperative radiation reveals that the main source of statistical spread of the output characteristics is the shot noise which is inherent to electron

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beams. The impact, which the shot noise produces, can be diminished by beam premodulation at radiation frequency [16–20]. Naturally, one could wonder what degree of beam premodulation is required for certain applications. This problem would be most prevalent for high-current generators with electron beams composed of ectons, individual portions of electrons that contain up to 10^{11} elementary charge carriers [32,33]. To estimate the shot noise accurately, one must take into account the complex structure of electron beams [34]. This is essential when solving the problem of coherent summation of electromagnetic oscillations from several short-pulse sources of radiation [5,6].

This paper considers the effect of shot noise on the generation of cooperative cyclotron radiation from a premodulated short beam of particles. We shall consider this issue by the example of an ensemble of nonisochronous electron-oscillators interacting with one another via the radiation field, used as one of the simplest models for the description of nonlinear generation of electromagnetic waves in CRMs [35–40]. The term "nonisochronous" is related to oscillators with amplitude-dependent frequencies [35]. It has been shown in [37,38] that in the absence of external action, the instability evolves in the ensemble of nonisochronous electron-oscillators, accompanied at the initial stage by an exponential growth of the radiated power and autophasing of electron-oscillators. This exponential growth is then suppressed due to nonlinearity, and the pulse of cooperative cyclotron radiation is formed [37]. The influence of nonisochronism on the peak power of cooperative radiation and the autophasing time of electron-oscillators was studied in detail by Vainshtein and colleagues [38]. However, the authors of [38] assumed that the beam had been premodulated at the radiation frequency and left out the effects related to shot noise caused by spread in positions and velocities of electrons [40]. The velocity spread is usually associated with the thermal nature of electron emission from the cathode surface. The position spread is originated from fortuitousness of the time moments of electron emission.

In our analysis of statistical properties of cooperative cyclotron radiation, the peak radiated power and autophasing time serve as random variables.

The paper's outline is as follows. First, we derive a system of equations describing the interaction of nonisochronous oscillators via the radiation field by the example of electron ensemble circulating in a uniform magnetic field. Further comes the consideration of statistical fluctuations in cooperative cyclotron radiation in the presence of shot noise from the ensemble of nonisochronous electron-oscillators with and without phase premodulation of charged particles. Particular attention is given to finding the autophasing time distribution function.

2. Cooperative cyclotron radiation

Let us consider the behavior of a weakly relativistic electron beam in a uniform magnetic field $\vec{H}$ directed along the OZ axis in the presence of the radiative energy loss. Particle velocity components perpendicular and parallel to the magnetic-field vector are denoted by $\vec{v}_{\perp k}$ and $\vec{v}_{zk}$, respectively. Then the behavior of particles is described by the equations of motion in the form

$$\begin{aligned} \dot{\vec{p}}_{\perp k} &= \frac{e}{c}\vec{v}_{\perp k}\times\vec{H}+\vec{F}_{\perp k}, \\ \dot{p}_{zk} &= F_{zk}. \end{aligned} \tag{1}$$

Here, $\vec{F}_k$ is the radiation reaction force acting on the kth particle from all particles and $\vec{p}_k$ is its momentum related to the velocity $\vec{v}_k$ as

$$\vec{p}_k=\frac{m\vec{v}_k}{\sqrt{1-v_k^2/c^2}}\approx m\vec{v}_k\left(1+\frac{v_{\perp k}^2+v_{zk}^2}{2c^2}\right). \tag{2}$$

If the beam size is less than the radiation wavelength $\lambda=2\pi mc^2/eH$, and the Coulomb repulsion force and induction fields can be neglected, then the radiation reaction force acting on each particle is given by the sum [41]

$$\vec{F}_k=e\sum_j\frac{2e}{3c^3}\ddot{\vec{v}}_j, \tag{3}$$

where $\frac{2e}{3c^3}\ddot{\vec{v}}_j$ is the radiation field induced by the jth particle.

Let us pay attention to an essential circumstance [41]: the expression for $\vec{F}_k$ is true if the radiative friction force is appreciably less than the Lorentz force $\frac{e}{c}\vec{v}_k\times\vec{H}$ acting on each particle. Otherwise, unphysical self-accelerated solutions may arise.

The requirement that the radiative friction force should be much less than the Lorentz force imposes limitation on the magnetic field strength (in the opposite case the considered theory is invalid). For single particle [41]:

$$H\ll\frac{m^2c^4}{e^3}\approx 6\cdot 10^{15}\text{ Gs}. \tag{4}$$

But in the case of a dense beam of coherently emitting particle with N_e electrons, mass $M=N_em$, and charge $Q=N_ee$ we can write by analogy with (4)

$$H\ll H_{cr}=\frac{M^2c^4}{Q^3}=\frac{m^2c^4}{N_ee^3}. \tag{5}$$

With the present-day acceleration facilities, dense beams with $N_e\sim 10^{10}$ electrons are available; substitution of $N_e\sim 10^{10}$ into (5) gives $H\ll$ 60 kGs. If this condition is violated, the equation set (1) with the force (3) is inapplicable.

Thus, if the radiation reaction force is much less than the Lorentz force and the beam size is less than the radiation wavelength, then the equations of motion describing the interaction between charged particles have the form:

$$\begin{aligned} \dot{p}_{\perp k} &= \frac{e}{c}\vec{v}_k\times\vec{H}+\frac{2e^2N_e}{3c^3}\ddot{\vec{v}}_{\perp}, \\ \ddot{\vec{v}}_{\perp} &= \frac{1}{N_e}\sum_k\ddot{\vec{v}}_{\perp k}. \end{aligned} \tag{6}$$

In the absence of energy losses through emission, the particles are in circular motion with cyclic frequencies [41]

$$\Omega_k=\frac{eH}{mc}\sqrt{1-\frac{v_k^2}{c^2}}\approx\frac{eH}{mc}\left(1-\frac{v_{\perp k}^2+v_{zk}^2}{2c^2}\right), \tag{7}$$

depending on $\vec{v}_{\perp k}$, which is responsible for nonisochronism of oscillations.

Using the approximate relation

$$\ddot{\vec{v}}_{\perp k}\approx-\Omega_k^2\vec{v}_{\perp k}\approx-\Omega^2\vec{v}_{\perp k}, \tag{8}$$

where

$$\Omega=\frac{eH}{mc}, \tag{9}$$

we shall write vector equations (6) in a component-wise fashion

$$\begin{aligned} \dot{v}_{xk} &= \Omega\left(1-\frac{1}{2}\frac{\vec{v}_{\perp k}^2+\vec{v}_{zk}^2}{c^2}\right)v_{yk}-\frac{2e^2\Omega^2N_e}{3mc^3}v_x, \\ \dot{v}_{yk} &= -\Omega\left(1-\frac{1}{2}\frac{\vec{v}_k^2+\vec{v}_{zk}^2}{c^2}\right)v_{xk}-\frac{2e^2\Omega^2N_e}{3mc^3}v_y, \\ \dot{v}_{zk} &= 0, \end{aligned} \tag{10}$$

where

$$\begin{aligned} \vec{v}_x &= \frac{1}{N_e}\sum_k\vec{v}_{xk}, \\ \vec{v}_y &= \frac{1}{N_e}\sum_k\vec{v}_{yk}. \end{aligned} \tag{11}$$