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Random versus periodic microstructures for elasticity of fibers reinforced composites

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ABSTRACT

Homogenization of effective elastic properties of fiber reinforced composites frequently supposed periodic arrangement of fibers because such microstructures allow analytical or low computational cost integrations. Hexagonal frame was often preferred than square one which is strongly anisotropic. In practical situations those periodical microstructures are not realistic. Real microstructures are often random or if they are periodic their boundaries don't fit with the periodic scheme. We studied with the help of finite elements samples that exhibit hexagonal arrangement of fibers embedded in a random distribution. Characteristic length scales of hexagonal area were extracted from observation of stress maps. Principal results are a short scale in which bulk and shear stresses become structured. On the other hand we nether reached a size large enough to observe local stress maps similar to those produced by a periodic model.

1. Introduction

Determination of the laws of macroscopic mechanical linear elasticity of composite materials built with longitudinally arranged fibers in a matrix (Fiber-reinforced composites) requires, in addition to knowledge of the laws of the behavior of each individual component, an averaging of local properties. This process is named homogenization. The most sensitive step in homogenization process is the localization of inclusions, fibers for material under consideration. In order to get analytical laws, most of the time, averaging was performed on periodic spatial distributions of the fibers. The representative volume element (RVE) was then reduced to an elementary cell surrounding a single inclusion [1]. Real microstructures are rarely perfectly periodic. They generally exhibit a random distribution of fibers [2], or a rough periodic distribution with gap between their real location and the periodic scheme. Therefore some numerical homogenizations were carried out [3,4] on material with random distribution of fibers. Boundary-layer effects on the effective response of fibers reinforced composites were analyzed in Ref. [5]. The authors focus on the modeling of the stress and strain in a domain of random composites. The distribution of the fibers in matrix is assumed random.

Usual periodic schemes were square cells or hexagonal cells. Square cells distribution presents a strong anisotropy characterized by two privileged directions. Therefore hexagonal periodic material is often selected to homogenization of elastic behavior [6]. The homogenization problem of a periodic composite with nonlinear hyperelastic constituents and debonded frictionless interfaces was studied in Ref. [7]. More recently, modeling of the elasticity of composite and carbon nanotubes fibers reinforced matrix were given in Refs. [8,9], respectively.

Looking at an inclusion located at a distance r from a test inclusion, this inclusion is in reality placed in a bin of sizes $r\delta r\delta\theta$. At short distance r the number of fibers in a bins is very sensitive to θ whereas at long distance it becomes independent or quasi isotopic. On the other hand the boundary conditions of hexagonal periodic distributions are not plane and consequently far from realistic situations.

A method to ensure realistic boundary condition consists in embedding the composite material in an outer region described by macroscopic laws [10–12]. The present approach is rather similar but in order to avoid building macroscopic constitutive law for the outer region it was a random composite material too. The outer

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region was build with similar fibers and matrix than the inner periodic materiel with a same volume fraction. Experiments were carried out with the help of a finite elements code on samples in which the sizes of the hexagonal inner region was varied. Results were compared to behavior of a single hexagonal cell as it was used in homogenization. The aim of this study is to determine characteristic sizes of the hexagonal region in which local behavior of stress field become similar to infinite periodic material.

A basic question is the efficiency of periodic model to predict effective properties of random or imperfectly periodic composites at microscopic scale. For that purpose we focus on characteristic sizes of hexagonal periodic area in which local behavior of stress field become similar to infinite periodic material.

Section 2 describes the methodology employed to generate numerical samples. Section 3 summarizes the mesh technique, boundary conditions, finite elements modeling and homogenization procedure. Section 4 presents local results on in-plane bulk and shear stresses that will be discussed in conclusion.

2. Generation of microstructures

Each microstructure, or sample, was an arrangement of 200 fibers distributed in parallel in a volume of square section. The fiber volume fraction was remained constant at$0.5 = 50\%$. Young's moduli and Poisson's ration were respectively for the matrix and the fibers:$E_m = 10Gpa$, $E_f = 1000Gpa$ and $\nu_m = \nu_f = 0.3$.

Four configurations were studied. They are sketched on Fig. 1. All configurations were built around a test fiber (in grey) located in the center of the cross section. Various circular areas, centered on the test fiber, were filled by a periodic hexagonal distribution of fibers. The rest of the section was then completed by a random distribution of fibers. These configurations are called Hexa7, Hexa19, …, Hexa61 in which numbers indicate the number of fiber hexagonally arranged.

In addition a sample so called random in which 199 fibers were randomly located around the test fiber was build, Fig. 2a. It could be imagine that it was a limit case with only one hexagonal inclusion.

Simulations were also performed on infinite periodic hexagonal fiber reinforced material which will be used as reference, Fig. 2b. As above mentioned, in such case the RVE is reduced to a single inclusion centered in a hexagonal cell of matrix, see Fig. 2b.

3. Homogenization and numerical procedure

3.1. Homogenization method

Effective properties of the different samples were provided by a finite element method that was still used for homogenization of effective properties of reinforced composites. Therefore, the macroscopic strain E and the macroscopic stress Σ are introduced, according to the definition of Hill. They derive from the microscopic strain ε and the microscopic stress σ obeying these equations:

$$E = \langle \varepsilon \rangle = \frac{1}{V} \int_V \varepsilon \, d\,V \qquad (1)$$

$$E = \langle \sigma \rangle = \frac{1}{V} \int_V \sigma \, d\,V \qquad (2)$$

with V denoting the domain occupied by the considered unit cell of composite materials.

The multi-scale character of the studied microstructures is used to carry out the averaging procedure. It should be noted that the periodic unit cells are valid representative volume elements (RVEs) of the considered microstructures. FE simulations were conducted on the RVE of each geometry in order to characterize their homogenized elastic properties under different boundaries conditions (see section 3.2). The RVEs were meshed with quadratic elements and the FE code Z-set (http://www.zset-software.com/) was used for the simulations.

For transverse isotropic problems, in the case of two phases, fibers f in a matrix m, the plane bulk moduli k_m and k_f and the plane shear moduli μ_m and μ_f of the matrix and fibers are related to the Young's moduli E_m and E_f and Poisson's ratios ν_m and ν_f as follows:

$$k_i = \frac{E_i}{2(1+\nu_i)(1-2\nu_i)}, \; \mu_i = \frac{E_i}{2(1+\nu_i)}, \; i = m, f \qquad (3)$$

3.2. Boundary conditions

Two classical boundary conditions, usually used in computational homogenization, were prescribed on the boundary of the domain [13]. For linear elasticity, these conditions are kinematic uniform boundary conditions (KUBC) and periodicity conditions (PBC). It should be noted that the minimal size required to estimate the effective properties is lower for periodic boundary conditions than for KUBC ones. So results converge faster with periodic boundary conditions. Terada et al. (2000) [14] claimed that the PBC provide the most efficient estimation among the class of admissible boundary conditions for statistically homogeneous media. They expressions are:

- KUBC is described by imposing the displacement $\underline{u}$ at point $\underline{x}$ which belongs to the boundary ∂V:

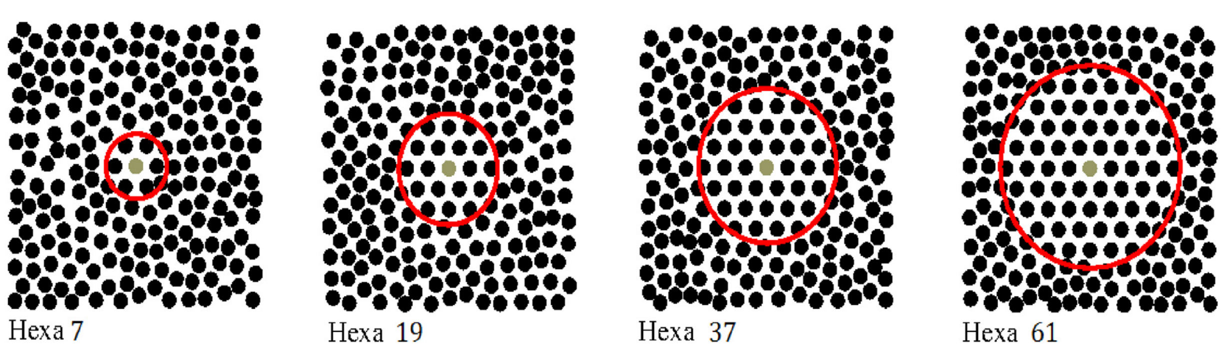

Fig. 1. Samples of configurations: Hexa7, Hexa19, Hexa37 and Hexa61.