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Finite element formulation of laminated beams with capability to model the thickness expansion

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ABSTRACT

This paper presents a static analysis of laminated beams by using a 6 degree-of–freedom hybrid type quasi-3D higher order shear deformation theory (HSDT). The governing equations are derived by employing the principle of virtual work and solved by means of Hermite-Lagrangian finite element method for laminated beams with several boundary conditions. A mixed interpolation, C^1 cubic Hermite and a C^0 linear Lagrange interpolation are used for the kinematic variables. Different types of shear strain shape functions were introduced *a priori* and in general manner to model the displacement field of the laminated beams. Convergence studies were performed in order to validate the HSDTs solved through finite element method and the results are compared with a Navier solution. Numerical results of the present generalized quasi-3D theory are also compared with FEM solutions predicted by other HSDT and with experimental results.

1. Introduction

Composite materials have numerous advantages compared to traditional metallic materials. However, laminated composite beam studies are not simple, and for example, a good model that can reproduce transverse shear stresses in the thickness direction is required, since laminated beams have considerable transverse stress fields. The classical theories for beams are inadequate in this case because they neglect transverse shear deformations. To account for this effect, higher order shear deformation theories (HSDTs) are necessary.

The first order shear deformation theory (FSDT) is an improvement over the Euler-Bernoulli theory. However, this theory has limited use because it assumes a constant transverse shear deformation through the thickness of the beam. To overcome the limitations of this theory, numerous HSDTs for laminated beams have been developed to include the effects of transverse shear deformations. Khdeir and Reddy [1] studied the bending of cross-ply laminated beams using the classical beam theory, FSDT and two HSDTs. A shear correction factor for laminated rectangular beams has been derived by Raman and Davalos [2].

Pagano [3] presented the elasticity solutions for laminated beams in cylindrical bending, and these results are widely used for comparison of closed-form solutions. Aydogdu [4] proposed a HSDT for the static and dynamic analysis of laminated plates and beams. A HSDT with third-order axial effects has been presented by Cook and Tessler [5]. Chen et al. [6] developed an analysis of laminated beams with a sinusoidal load using a modified couple-stress theory. Zigzag models for the bending analysis of composite beams have been presented by Arya et al. [7] and Icardi [8]. Mantari and Canales [9] developed a quasi-3D HSDT theory for the bending analysis of simply supported laminated beams subjected to sinusoidal, uniformly distributed, linearly varying and point loads. Mantari and Yarasca [10] modeled functionally graded composite beams using a quasi-3D hybrid theory that includes thickness stretching effects. Vo and Thai [11] developed a shear deformation theory for the static analysis of composite beams. An assessment of zigzag theories for the analysis of laminated beams has been presented by Icardi and Sola [12].

Further developments of Finite Element Analysis (FEA) to consider HSDTs have been performed. Lo et al. [13] presented a HSDT model for composite beams and used it to obtain closed-form and finite element solutions. Sciuva et al. [14] developed beam elements for the analysis of composite beams using a zig-zag theory. Kant and Manjunath [15] developed a beam finite element for the analysis of multilayered beams with a sinusoidal loading, and

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presented a HSDT that estimated transverse stresses integrating the equations of equilibrium to get accurate results [16]. An analysis of laminated beams using a sine finite element has been presented by Vidal and Polit [17]. Vo and Thai [18] developed a finite beam element to study the vibration and buckling of composite beams. Thin-walled beams are structural members of great importance, and analyses of this structure is given in Refs. [19–23]. To model the stresses with a smoothness via FEM, Hermite-cubic polynomial was used to ensure C^1 continuous elements. It is important to remark that the generation of Hermite polynomials is described in many mathematical texts [24–26].

A unified formulation known as Carrera's Unified Formulation (CUF) has been developed and applied for the analysis of laminated beams in quasi-3D manner. Carrera et al. [27] presented beam elements with arbitrary cross-sections using this unified formulation. Further progress has been done to develop a static analysis of laminated composite beams in Catapano et al. [28]; to model laminated structures with fibers, matrices and multilayers in Carrera et al. [29]; to consider trigonometric, exponential and zig-zag theories in Carrera et al. [30] and Filippi and Carrera [31], and to analyze composite structures using a multi-line approach in Carrera and Pagani [32]. The unified formulation is described in Refs. [33,34].

Beam theories have been used for special applications and to analyze engineering structures. Milazzo and Orlando [35] presented a beam finite element for the analysis of magneto-electro-elastic multilayered composite structures. Zemanová et al. [36] developed a finite element approach to model laminated glass beams under finite strain. Apedo et al. [37] modeled inflatable beams made from orthotropic materials taking into account geometric nonlinearities and the inflation pressure. Kapuria and Alam [38] developed a beam finite element for the dynamic analysis of piezoelectric beams. Vidal and Polit [39] presented an analysis of thermo-mechanical laminated beams using a finite element approach. Roche and Accorsi [40] developed a finite element model to analyze laminated beams taking into account delaminations. Nonlinear analyses of piezoelectric fiber reinforced laminated beams has been developed by Shen et al. [41] and Mareshi et al. [42].

In this paper, a hybrid type quasi-3D HSDT for the bending analysis of laminated beams is solved by Hermite-Lagrangian finite element technique. Infinite quasi-3D hybrid type (polynomial, non-polynomial, and hybrid) HSDTs solved by FEM can be derived by using the present generalized theory for beams. The beam governing equations are derived by employing the principle of virtual work for laminated beams subjected to transverse load for simply supported, clamped-clamped, clamped-free and clamped-simple support boundary conditions. The results are compared with analytical solutions and with experimental results. Convergence studies are performed in order to guaranty the finite element technique adopted to solve the present hybrid type quasi-3D HSDT for beams.

2. Analytical modeling

2.1. Beam under consideration

A cross-ply laminated beam of length L, width b and a total thickness h is considered in the present analysis. The beam occupies the following region:

$$0 \le x \le L; \quad -b/2 \le y \le b/2; \quad -h/2 \le z \le h/2$$

The displacements are assumed to be small, and the body forces are neglected. The beam is subjected to lateral load only, and two dimensional constitutive laws are used (x and z).

2.2. Theoretical displacement field

The displacement field satisfying the free surface boundary conditions of transverse shear stresses vanishing at a point $(x, \pm h/2)$ on the top and bottom surfaces of the beam, is given as presented in Ref. [9]:

$$\begin{aligned} u(x,z) &= u_0 + z\left[-\frac{\partial w_0}{\partial x} + q^*\frac{\partial \theta}{\partial x} + y^*\varphi\right] + f(z)\varphi \\ w(x,z) &= w_0 + g(z)\theta \end{aligned} \tag{1}$$

where u and w are the displacement components in the X and Z axis respectively. In addition, u_0,w_0, θ and φ are four unknown displacements of midplane of the beam. The constants y^* and q^* are obtained by considering the criteria to reduce the number of unknowns in HSDTs as in Reddy and Liu [43]. They are as a function of the shear strain shape functions, $f(z)$ and $g(z)$, i.e. $y^* = -f'\left(\pm\frac{h}{2}\right)$ and $q^* = -g(\pm h/2)$.

For deriving the equations, small elastic deformations are assumed, i.e. displacements and rotations are small, and obey Hooke's law. The starting point of the present generalized quasi-3D HSDT solved through Hermite-Lagrangian finite element technique is the 3D elasticity theory [44]. The strain-displacement relations, based on this formulation, are written as follows:

$$\begin{aligned} \varepsilon_X &= \varepsilon_X^1 + z\varepsilon_X^2 + f(z)\varepsilon_X^3 \\ \varepsilon_Z &= g'(z)\varepsilon_Z^6 \\ \gamma_{XZ} &= \gamma_{XZ}^1 + g(z)\gamma_{XZ}^4 + f'(z)\gamma_{XZ}^5 \end{aligned} \tag{2}$$

where

$$\begin{aligned} &\varepsilon_X^1 = \frac{\partial u_0}{\partial x} \quad \varepsilon_X^2 = -\frac{\partial^2 w_0}{\partial x^2} + q^*\frac{\partial^2\theta}{\partial x^2} + y^*\frac{\partial\varphi}{\partial x} \quad \varepsilon_X^3 = \frac{\partial\varphi}{\partial x} \\ &\varepsilon_Z^6 = \theta \\ &\gamma_{XZ}^1 = q^*\frac{\partial\theta}{\partial x} + y^*\varphi \quad \gamma_{XZ}^4 = \frac{\partial\theta}{\partial x} \quad \gamma_{XZ}^5 = \varphi \end{aligned} \tag{3}$$

In vector form this can be expressed as:

$$\varepsilon^T = \{\, \varepsilon_X \quad \varepsilon_Z \quad \gamma_{XZ} \,\} \tag{4}$$

$$\varepsilon = \varepsilon^1 + z\varepsilon^2 + f(z)\varepsilon^3 + g(z)\varepsilon^4 + f'(z)\varepsilon^5 + g'(z)\varepsilon^6 \tag{5}$$

where

$$\begin{aligned} &\varepsilon^1 = \begin{Bmatrix} \varepsilon_X^1 \\ 0 \\ \gamma_{XZ}^1 \end{Bmatrix} \quad \varepsilon^2 = \begin{Bmatrix} \varepsilon_X^2 \\ 0 \\ 0 \end{Bmatrix} \quad \varepsilon^3 = \begin{Bmatrix} \varepsilon_X^3 \\ 0 \\ 0 \end{Bmatrix} \\ &\varepsilon^4 = \begin{Bmatrix} 0 \\ 0 \\ \gamma_{XZ}^4 \end{Bmatrix} \quad \varepsilon^5 = \begin{Bmatrix} 0 \\ 0 \\ \gamma_{XZ}^5 \end{Bmatrix} \quad \varepsilon^6 = \begin{Bmatrix} 0 \\ \varepsilon_Z^6 \\ 0 \end{Bmatrix} \end{aligned} \tag{6}$$

The stresses can be written using the same approach

$$\sigma^T = \{\, \sigma_X \quad \sigma_Z \quad \tau_{XZ} \,\} \tag{7}$$

The linear constitutive relations become:

$$\sigma = Q\varepsilon \tag{8}$$

where