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Size dependent nonlinear vibration of the tensioned nanobeam based on the modified couple stress theory

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ABSTRACT

This paper presents a nonlinear vibration analysis of the tensioned nanobeams with simple–simple and clamped–clamped boundary conditions. The size dependent Euler–Bernoulli beam model is applied to tensioned nanobeam. Governing differential equation of motion of the system is obtain by using modified couple stress theory and Hamilton's principle. The small size effect can be obtained by a material length scale parameter. The nonlinear equations of motion including stretching of the neutral axis are derived. Damping and forcing effects are considered in the analysis. The closed form approximate solution of nonlinear equations is solved by using the multiple scale method, a perturbation technique. The frequency-response curves of the system are constructed. Moreover, the effect of different system parameters on the vibration of the system are determined and presented numerically and graphically. The size effect is significant for very thin beams whose height is at the nanoscale. The vibration frequency predicted by the modified couple stress theory is larger than that by the classical beam theory. Comparison studies are also performed to verify the present formulation and solutions.

1. Introduction

Nanostructures have gained considerable interest in scientific research studies since the discovery of carbon nanotubes by Iijima [1]. Due to their superior electrical and mechanical properties, nanostructures are used by scientists in various areas such as sensor technologies, composites and electromechanical systems.

The classical continuum theory, independent from size, is unable to model the nanostructures, where as non-classical continuum theories, dependent of size, are used to model static and dynamic behavior of micro and nanostructures. These theories include the modified couple stress theory [2], the nonlocal elasticity theory [3], the strain gradient theory [4], the surface elasticity [5], the micropolar theory [6]. The modified couple stress theory was initially applied by Park and Gao [7] to static deformation analysis of a microcantilever Euler–Bernoulli beam subjected to a point load.

The literature regarding the vibrations of microbeams based on modified couple stress theory is quite large. For example, Works of Ma et al. [8] deals with the microstructure dependent model for the Timoshenko beam based on the modified couple stress theory for solving the static bending and free vibration of a simply-supported microbeam. Kong et al. [9] solved analytically free vibration problem of beams based on the modified couple stress and Euler–Bernoulli beam theory. Fu and Zhang [10] developed a size dependent Timoshenko beam model based on the modified couple stress theory for investigating the mechanical behavior of microtubules. Wang [11] studied the size dependent free vibration of fluid conveying microtubes using the modified couple stress theory. Kahrobaiyan et al. [12] carried out a study related with the size dependent mechanical behaviors of atomic force microscope microcantilever based on the modified couple stress theory. Xia et al. [13] established a nonlinear non-classical Euler–Bernoulli beam model by using the modified couple stress theory for investigating the static bending, postbuckling and free vibration of microscale beams. Şimşek [14] studied the vibration of an embedded microbeam under action of a moving microparticle based on the modified couple stress theory. Asghari et al. [15] considered the size dependent Timoshenko beam model based on the modified couple stress theory for investigating the static and free vibration behaviors of a hinged-hinged microbeam. Also this author and its coauthors [16] considered the size effects in Timoshenko beams based on the modified couple stress theory. Ahangar

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et al. [17] investigated the size dependent vibration behavior of a microbeam conveying fluid using the modified couple stress theory. Akgöz and Civalek [18] address stability problem of micro-sized beam based on the strain gradient elasticity and modified couple stress theories. These studies were pursued in Refs. [19–28] where developed a size dependent microbeam model on the basis of the modified couple stress theory.

Although the literature regarding the vibration analysis of microbeams based on modified couple stress theory is quite large, the literature regarding the vibration analysis of nanobeams using modified couple stress theory is rather limited. For example, Ke and Wang [29] investigated vibration and instability of the embedded fluid conveying double walled carbon nanotubes (DWCNT) based on modified couple stress theory and the Timoshenko beam theory. Abdi et al. [30] considered the size effect on the static pull-in instability of electrostatic nanocantilevers in the presence of van der Waals forces based on modified couple stress theory. Akgöz and Civalek [31] studied the stability analysis of carbon nanotubes based on modified couple stress theory and Euler–Bernoulli beam theory. Arani et al. [32] studied the nonlinear free vibration and instability of fluid conveying double walled boron nitride nanotubes embedded in viscoelastic medium based on the modified couple stress theory and Timoshenko beam theory to address the size effect of clamped–clamped nanotubes. Fakhrabadi et al. [33] investigated the effects of fluid flow on the static and dynamic pull-in instabilities of carbon nanotubes conveying viscous fluid based on the modified couple stress theory. Another study of Fakhrabadi et al. [34] deals with the size dependent mechanical behaviors of the cantilever and doubly clamped carbon nanotubes under electrostatic actuation using the modified couple stress theory. Zeighampour and Beni [35] considered the size dependent vibration of DWCNT conveying fluid in the presence of the van der Waals force on the base of modified couple stress theory. Beni et al. [36] studied the effect of intermolecular van der Waals force on the size dependent pull-in of nanobridges and nanocantilevers based on modified couple stress theory. Miandoab et al. [37] derived an analytical solution of polysilicon nanobeam based on modified couple stress theory and Eringen's nonlocal elasticity theories. Bağdatli [38] studied nonlocal nonlinear vibrations of Euler–Bernoulli nanobeams with various supports condition. Recent contributions on the nonlocal behavior of nanostructures have been provided also for functionally graded (FG) materials; see e.g. Refs. [39–41]. In addition to that, a number of studies were also conducted to investigate the random composite beams under torsion [42], the bending of nanobeams [43] and Timoshenko nanobeams [44]. Nonlinear vibrations of an Euler–Bernoulli nanobeam resting on an elastic foundation [45] and Pasternak foundation [46] are studied using nonlocal elasticity theory.

We examine the literature presented in the above, it can be seen that almost all researchers have so far studied buckling, static and vibration analysis of micro and nanobeams on the base of modified couple stress theory. The nonlinear transverse vibrations of a tensioned nanobeam under two boundary conditions are studied using Eringen nonlocal elasticity theory [47]. To the best of the authors' knowledge, size dependent nonlinear vibration of tensioned nanobeams on the base of the modified couple stress theory seems to be nonexistent. The main objective of the current study is to fill this gap. The nonlinearity of the problem is obtained by including stretching of the neutral axis that introduces cubic nonlinearity into the equations. Nonlinear frequency-response curves are drawn for nanobeams with two different end conditions which are named as simple–simple and clamped–clamped nanobeams.

2. The modified couple stress theory

In the modified couple stress theory was initially proposed by Yang et al. [2], the strain energy density is a function of both strain tensor and curvature tensor. Therefore, the strain energy of a deformed isotropic linear elastic body occupying a volume Ω is given as

$$U = \frac{1}{2}\int_{\Omega}\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right)dV \tag{1}$$

in where σ_{ij} and ε_{ij} are the stress tensor and strain tensor, respectively. m_{ij} is the deviatoric part of the couple stress tensor, χ_{ij} is the symmetric curvature tensor. These tensor are expressed as

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

$$m_{ij} = 2\mu l^2\chi_{ij} \tag{4}$$

$$\chi_{ij} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right) \tag{5}$$

where u_i and δ_{ij} are the displacement vector and the Kronecker delta, respectively. l is the material length scale parameter and θ_i is the rotation vector that can be defined as

$$\theta_i = \frac{1}{2}e_{ijk}u_{k,j} \tag{6}$$

where e_{ijk} is the permutation symbol. λ and μ are the Láme's constants that are defined as

$$\lambda = \frac{\mathrm{E}\upsilon}{(1+\upsilon)(1-2\upsilon)}, \quad \mu = \frac{\mathrm{E}}{2(1+\upsilon)} \tag{7}$$

where υ is Poisson's ratio, μ is shear modulus and E is Young's modulus.

3. Equation of motion

In this study, vibration of tensioned nanobeam is carried out on the base of size dependent Euler–Bernoulli beam model. Schematically representation of the system is shown in Fig. 1. Two types of boundary conditions which are simple–simple and clamped–clamped are considered in this work.

For the system shown in Fig. 1, y^* denotes the transverse displacement of the beam section between supports. L is the length of the beam. The Lagrange equation of the proposed model is given in Eq. (8), t^* is the time. ρA is the mass per unit length, N^* is the axial force, EA is longitudinal rigidity, EI is flexural rigidity and P^* is the axial tension force.

$$\begin{aligned}£ = {} & \frac{1}{2}\int_0^L \rho A\left(\frac{\partial y^*}{\partial t^*}\right)^2 dx^* - \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 y^*}{\partial x^{*2}}\right)^2 dx^* \\ & - \frac{1}{2}\frac{E}{2(1+\nu)}Al^2\int_0^L\left(\frac{\partial^2 y^*}{\partial x^{*2}}\right)^2 dx^* - \frac{1}{2}\int_0^L N^*\left(\frac{\partial y^*}{\partial x^*}\right)^2 dx^* \\ & - \frac{1}{2}\int_0^L P^*\left(\frac{\partial y^*}{\partial x^*}\right)^2 dx^*\end{aligned} \tag{8}$$